A FORTRAN 77 PROGRAM

FOTEL

USER'S MANUAL

SEPARATE OR SIMULTANEOUS RADIAL-VELOCITY AND LIGHT-CURVE SOLUTIONS

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FOREWORD

Fortran 77 program FOTEL was written and is being gradually developed by Dr. Petr Hadrava from the Astronomical Institute of Academy of Sciences in Ondřejov. Its early version is described in Hadrava (1990). I have been using this program since the early times of its develeopment and, realizing the potential of it, I annoyed Dr. Hadrava with frequent requests for further and further improvements of the code. Dr. Hadrava indeed did these improvements but not always had enough time to fully record and explain all the new options in User's manual which he provides with the program.

Becoming gradually a rather experienced user of FOTEL (see, e.g. Harmanec and Scholz 1993 or Harmanec et al. 1997 for some more sophisticated applications), I finally decided to write my own – user-oriented – manual to share my experience with others (and also to keep record of what I know now for my further reference).

Availability of the software

Source code of the FOTEL and a user guide are available from the personal web page of Dr. Petr Hadrava

http://www.asu.cas.cz/˜had/had.html

Interested users may also obtain my latest modification of the program with a few improvements, denoted

FOT2003.F (Unix) & **FOT2003.FOR (DOS),**

which fully comply with the description provided in this manual.
It is available via anonymous

ftp astro.mff.cuni.cz/hec/fotel

both in DOS and Unix versions.

Do not worry to use it. I am proceeding in a close collaboration with Dr. Hadrava who also has this version from me and will start any possible future modifications from it.

Note that the program is freely available to all interested colleagues upon the condition that its usage and the authorship of Dr. P. Hadrava will be acknowledged in any publication based on solutions obtained with it, for instance, by a reference to Hadrava's (1990) paper.

I strongly recommend you to notify Dr. P. Hadrava via e-mail (had@sunstel.asu.cas.cz) about your becoming a user of this software. Your e-mail address will be registered and you will be notified of any future improvement or extension of the software package.

Contents

1 PURPOSE, GENERAL BACKGROUND AND LITERATURE

Probably the most widely used of the programs for the light-curve solution is the WD program published by US astronomers R.E. Wilson and E.J. Devinney – see, e.g., Wilson a Devinney (1971). The program was later generalized to include a disk around one component, modelled as a rotationally flattened ellipsoid (Wilson 1974). The original program is not particularly user-friendly and was modified in several places (see, e.g. the well-known Binary Maker which is based on it). The WD program is strictly based on Roche geometry and allows also modelling of the light curves of ellipsoidal variables. Dr. R.E. Wilson (priv.com.) has a new version with a number of other improvements.

Other well-known programs are, for example, **LIGHT2** (Hill 1979), **EBOP** by Etzel and Bopp or the most sophisticated **BINSYN suite** of programs by A.P. Linnell, combined with disk models by I. Hubeny (Linnell 1984, Linnell a Hubeny 1994).

Finally, there is **FOTEL** (Hadrava 1990), to be disussed in detail here.

Program FOTEL (an acronym for FOTometric ELements) was written and gradually upgraded by an Ondřejov astronomer Dr. Petr Hadrava. It is one of the few existing programs which allow a simultaneous solution of light and RV curves, thus deriving critical elements like the time of periastron passage, longitude of periastron or orbital eccentricity consistently and more reliably.

Available versions of FOTEL *are not* based on Roche geometry (although the Roche model version is “under construction” and discussed in Hadrava’s manual to the program) but on the model of triaxial ellipsoids.

Tests on real data show that this approximation is sufficient in the majority of real situations considering other, still unsolved problems. If desirable, one can keep the radius of the Roche-lobe filling star fixed at appropriate radius and the convergency can be allowed only for the radius of the other star and all other required parameters.

1.1 Basic advantages of FOTEL:

Here are some reasons why FOTEL is to be preferred for a number of tasks:

1. *A consistent modelling of a hierarchical triple system* :

   System is assumed to be composed from a spectroscopic or eclipsing binary and a distant third body revolving around a common centre of gravity with the close pair. Solution can include orbital elements of the third body. The brightness of the third body is assumed constant but its effect on the light curve is modelled and its value can be obtained from the solution provided the close pair is an eclipsing binary.

2. *A consistent modelling of the periastron advance if present* It is posible to include the apsidal advance $\dot{\omega}$ among elements of the solution which gives much better result than the usual modelling of times of minima only.

3. *Secular change of the orbital period can be modelled*

   A time derivative of the orbital period, $\dot{P}$, can be included among the elements obtained during the solution; the user can decide between a monotonic change (characterized by a parabolic trend in the $O-C$ diagram or a general change of the orbital period.

4. *Secular changes* of the semiamplitude of the RV curve and/or brightness of the binary in maximum light can also be included among the elements of the solution.

5. *Program consistently uses individual data points* and their times of observations, not normal points in a phase diagram.

6. *Simultaneous solution of light curves in several passbands*

   It is easy to derive solution simultaneously for data from various sources and from different passbands. Individual zero points are then automatically derived for each data set, both in photometry and RVs from different spectrographs. If desired, however, the program can be instructed to derive a joint zero point for several data sets.

7. *Modelling of changes of stellar radii with orbital phase*

   It is also possible to derive periastron and apastron radii and their smooth change for close binaries in highly eccentric orbits.

8. *Good error estimates*

   Errors of the elements of the solution are calculated with the help of a covariance matrix and the output of the program also contains information how much the individual elements are correlated with each other.

9. *All important epochs are given*

   Output of the program provides the user with values of all important time instants: the epochs of the conjunctions and mid-eclipses and RV maxima and minima.

1.2 Limitations of the current version of FOTEL

1. A simultaneous convergency of a maximum of 20 different elements is possible.

   While sufficient in most of situations, this can lead to certain problems when one treats a complicated system with many various data sets, both photometric at several passbands and RVs, in which apsidal advance, change of the orbital period and a third body are all present.

2. Current version of the program allows a simultaneous solution of the light curve in 10 different passbands at maximum.

3. A maximum of 30 different zero points can be treated.

4. Available versions of the program do not work in the regime of a dynamical allocation of computer memory which implies also a limitation on the maximum number of observations (5000 in the latest version). However, this can easily be enlarged by simply changing the value of one parameter and the dimension declarations in the program.

5. FOTEL should be able to derive elements for ellipsoidal variables according to its description but my experience is that this part of the program does not work properly. The well-known Wilson and Devinney (1971) program should be preferred in such cases.

6. FOTEL is designed to use only linear limb darkening although various more sophisticated models, based on real model atmospheres are described in the astronomical literature – see, e.g., Claret (2000). However, it is easy to verify via solutions based on widely different values of limb darkenings that for a vast majority of real binaries, solutions are quite insensitive to the accurate values of limb darkening.

1.3 Some relations, numerical method and other useful info about FOTEL:

The equations on which FOTEL solutions are based are explained in some detail in Hadrava's User Guide. I only mention some of them, which I consider the most important ones for the understanding what FOTEL does; the users are referred to Hadrava's text for the rest.

For a general case of a binary with variable orbital period and with the light-time effect due to motion in orbit with a distant third star, the mean anomaly is not defined by the usual simple relation but by the following formula

$$M = 2\pi\frac{t - T_0 - \Delta t}{P_0}\left(1 - \frac{t - T_0 - \Delta t}{2P_0}\left(\dot{P}_0 - k\frac{t - T_0 - \Delta t}{P_0}\right)\right) , \tag{1}$$

where T_0 is the reference time of the periastron passage defining phase zero which corresponds to instantaneous values of the orbital period P_0 and its change $\dot{P}_0$. The symbol Δt denotes the correction for the light-time effect, given by

$$\Delta t = \frac{P' K_{1+2}}{2\pi c}(1 - e'^2)^{3/2}\frac{\sin(\omega' + v')}{1 + e'\cos(v')} , \tag{2}$$

where K_{1+2} denotes the semiamplitude of the RV curve of the centre of gravity of the binary around the joint centre of gravity with the third body, c is the speed of light in vacuum and the quantity k is given by

$$k = \frac{1}{3}(P_0\ddot{P}_0 - 2\dot{P}_0^2)\ . \tag{3}$$

Elements with a prime correspond to motion of the centre of gravity of the binary around the common centre of gravity with the third body.

The instantaneous value of the orbital period $P(t)$ is then obviously given by

$$\begin{aligned} \frac{1}{P(t)} &= \frac{1}{2\pi}\frac{dM}{dt} \\ &= \frac{1}{P_0}\left(1 - \frac{t - T_0 - \Delta t}{2P_0}\left(2\dot{P}_0 - 3k\frac{t - T_0 - \Delta t}{P_0}\right)\right) \end{aligned} \tag{4}$$

and for its instantaneous change one gets

$$\dot{P}(t) = -\frac{P^2(t)}{2\pi}\frac{d^2M}{dt^2} = \frac{P^2(t)}{P_0^2}\left(\dot{P}_0 - 3k\frac{t - T_0 - \Delta t}{P_0}\right)\ . \tag{5}$$

If one needs to describe the run of deviations in the $O-C$ diagram, its is necessary to invert equation (1). One obtains

$$t = t_0 + \left(\frac{M}{2\pi}\right)P_0 + \frac{1}{2}P_0\dot{P}_0\left(\frac{M}{2\pi}\right)^2 + k'\left(\frac{M}{2\pi}\right)^3\ , \tag{6}$$

where

$$k' = \frac{P_0}{6}(P_0\ddot{P}_0 + \dot{P}_0^2) = \frac{P_0}{2}(k + \dot{P}_0^2)\ . \tag{7}$$

A special case of a secular period change which is particularly important from observer's point of view is that one which corresponds to a parabolic shape of the $O-C$ deviations considered as a function of epoch E. Times of minima are defined then by

$$T = T_0 + P_0 \cdot E + a \cdot E^2, \tag{8}$$

where P_0 is the instantaneous period at time of reference minimum T_0, i.e. for $E = 0$. Let us understand epoch E as *a real number* characterizing cycle and phase, then T will be time corresponding to the respective phase in given cycle. The instantaneous period is then obviously given by

$$P = \frac{\mathrm{d}T}{\mathrm{d}E} = P_0 + 2aE \tag{9}$$

and its instantaneous changee $\dot{P}$ is

$$\dot{P} = 2a\frac{\mathrm{d}E}{\mathrm{d}T} = \frac{2a}{P}. \tag{10}$$

This shows that for the quadratic ephemeris the period varies linearly with epoch but not with time. For positive a the rate of period change is decreasing with time.

This is, however, the situation which occurs during the usually observed later stages of mass exchange after the role of the components has interchanged. The quadratic ephemeris is, therefore a very good description of period variations in similar situations.

As already mentioned, one can instruct FOTEL to model this particular case of the period change. From (9) and (10) it follows that

$$P(t) \cdot \dot{P}(t) = konst.\ , \tag{11}$$

so that the expression for the mean anomaly (1) adopts the form

$$M = 4\pi\frac{t - t_0 - \Delta t}{P_0}\left(1 + \sqrt{1 + 2\dot{P}_0\frac{t - t_0 - \Delta t}{P_0}}\right)^{-1}\ . \tag{12}$$

As far as the secular changes of other elements like ω, semiamplitudes of RV curves K_j, $(j = 1, 2)$ or brightness of the system at maximum light are concerned, they are derived on assumption of *a linear* change, i.e.

$$\omega(t) = \omega_0 + \dot{\omega}_0(t - t_0 - \Delta t) \tag{13}$$

and similarly for other elements.

Program FOTEL uses a linear limb darkening u_λ, based on a simple model atmosphere. The limb darkening describes the change of intensity at given wavelength in the direction to the observer in dependence on the angle ϑ between the normal to the stellar surface at given point and the line of sight. The decrease of the intensity from the centre of stellar disk to the limb in wavelength λ can be written as a function of ϑ in the usual approximation as

$$I_\lambda(\vartheta) = I_\lambda(0)(1 - u_\lambda + u_\lambda \cos\vartheta). \tag{14}$$

This means that outside the eclipses one can write the sum of light contributions from the disk of a spherical star with a relative radius r as an integral of contributions from an infinitesimal annulus ring with a radius x a thickness $\mathrm{d}x$ as

$$L_\lambda = \int_0^r 2\pi x I_\lambda(\vartheta)\mathrm{d}x = \int_0^r 2\pi x I_\lambda(0)(1 - u_\lambda + u_\lambda \cos\vartheta)\mathrm{d}x. \tag{15}$$

The first two expressions can be integrated directly. This is also true for the third one considering that it follows from geometry that $x = r\sin\vartheta$ a therefore $\mathrm{d}x = r\cos\vartheta\mathrm{d}\vartheta$. One obtains

$$\begin{aligned} L_\lambda &= I_\lambda(0)\left(\pi r^2 - \pi r^2 u_\lambda + 2\pi r^2 u_\lambda \int_0^{\frac{\pi}{2}} \sin\vartheta\cos^2\vartheta\mathrm{d}\vartheta\right) \\ &= I_\lambda(0)\left(\pi r^2 - \pi r^2 u_\lambda + \frac{2}{3}\pi r^2 u_\lambda\right) \\ &= \pi r^2 I_\lambda(0)\left(1 - \frac{u_\lambda}{3}\right). \end{aligned} \tag{16}$$

Probably the best currently available values of the coefficients of the limb darkening can be found in the tables published by Claret (2000) for the wavelengths corresponding to passbands of the $UBVRI$ and $uvby$ photometric systems.

Typical values of these coefficients are abound 0.8–0.9 for cool stars of spectral classes K a M, 0.5–0.6 for A0 stars and 0.25–0.35 for B0 stars.

One should keep in mind that the shape of the primary and secondary is characterized by triaxial ellipsoids with semiaxes a_j, b_j a c_j, $j = 1, 2)$. The effective radius of $j-$th component, r_j, can then be approximated as

$$r_j = \sqrt[3]{a_j b_j c_j}. \tag{17}$$

Considering the geometrical projections one can write the effective radius outside eclipses as a function of true anomaly v as

$$r_j(v)^2 = [a_j^2 b_j^2 \cos^2 i + (b_j^2 + (a_j^2 - b_j^2)\cos^2(\omega + v))c_j^2 \sin^2 i]^{\frac{1}{2}}. \tag{18}$$

The instantaneous relative flux to a unit space angle along the line of sight is then given by

$$L_{\lambda,j} = \left(\frac{r_j(v)}{r_j}\right)^2 10^{-0,4m_{\lambda,j}}, \tag{19}$$

where $m_{\lambda,j}$ is the magnitude of $j-$th component at given wavelength at maximum light.

A third body – if present – is modelled as a spherical star so that if its radius r_3 is not zero, one has

$$L_{\lambda,3} = 10^{-0,4m_{\lambda,3}}. \tag{20}$$

The user has two basic options. In both cases, it is desirable to edit input data in such a way that **observations in different passbands and from different sources are clearly distinguished**.

1. *A free convergency of colour magnitudes*

   If this option is chosen, the program derives the magnitudes of the whole system at maximum light m_λ for all passbands and all identified data subsets separately and also returns values of

$$m_{\lambda,j} - m_\lambda = -2.5 \log \frac{L_{\lambda,j}}{L_{\lambda,1} + L_{\lambda,2} + L_{\lambda,3}} \tag{21}$$

   (j=1,2,3) on the output. From them, it is easy to derive the relative luminosities of the components $L_{\lambda,j}$ and also the colour indices of both or all three stars. If one uses data reduced to some well-defined standard photometric system, the colour indices can be individually dereddened and can provide a photometric estimate of the spectral types or effective temperatures of the stars. A good mutual agreement of the amount of reddening derived individually for each star represents a useful test of the reliability of the solution.

2. *Convergency of effective temperatures*

   It is also possible to obtain the effective temperatures of the components directly, as the elements of the solution, derived on the usual premise that the ratio of luminosities in all considered effective wavelenghts of photometric passbands corresponds to that of the respective blackbodies. Magnitudes of the components are then given by

$$m_{\lambda,j}(v) = -2,5 \log \left(r_j(v)^2 \frac{B_\lambda(T_j)}{B_\lambda(T_0)} \right) + \triangle m_j, \tag{22}$$

   where T_0 is the effective temperature of a reference star – either of the comparison star used if the input photometric data are magnitude differences relative to this comparison – or of a star of spectral class A0, the values for which are built in FOTEL.

   Note that in this regime one can also allow convergency of the value of interstellar extinction for calibrated photometry and on the assumption of some defined law of interstellar extinction as a function of wavelength.

The geometry of the eclipse in FOTEL is simplified to the eclise of two *spherical bodies* with relative radii $r_1(v)$ a $r_2(v)$.

The magnitude of the whole system at given wavelength is then given as

$$m_\lambda(v) = -2,5 \log(L_{\lambda,1}(v) + L_{\lambda,2}(v) + L_{\lambda,3}(v) - \triangle L_\lambda(v)), \tag{23}$$

where $\triangle L_\lambda(v)$ stands for the (complicated) functional dependence of the light change during the eclipse.

Reflection can also be considered and treated in FOTEL.

FOTEL uses the simplex method described by Kallrath and Linnell (1987) to find out a minimum of the sum of squares of $O-C$ in the multidimensional space for **non-linear parameters $\bar{p}$**.

This method first derives the sum of squares of $O-C$ in three diferent points around the input parameters and then tries to replace the point with the largest sum via four possible operations:

A. Mirror reflection – A new point is chosen to lie on the opposite side relative to the line joining the two remaining points with lower sums of squares and at the same distance from it.

B. Division – A new point is chosen on the same side with respect to the line joining the two better points but at one half of the distance of the original point from it.

C. Multiplication – A new point is chosen on the same side as the original one but at twice larger distance from it than the original one.

D. Contraction – Two points with larger sums of squares are moved to one half of the distance from the best point to it.

Program FOTEL can show the convergency on the computer screen and identifies individual operations with letters A to D as described above.

The number of transformations of the simplex during each iteration is ten times larger that the number of non-linear elements of the solution. Note that elements with a linear functional dependence (like the semiamplitude of the RV curve of the primary) are calculated directly by the least-square method as described in Hadrava's FOTEL User's Guide and **are not** included in the above number.

FOTEL internally transforms certain elements having a limited range of their admissible values (like eccentricity which can only vary from 0 to 1) to variables defined over the whole range of real numbers to prevent convergency problems. Such an approach was first implemented by the late Dr. Jiří Horn in his program SPEL for RV-curve solution. In particular, the original values E_n of such elements are transformed via the following logarithmic transformation

$$\begin{aligned} p_n &= \ln\left((E_n - E_{min}) \cdot (E_{max} - E_n)^{-1}\right) && \text{for} \quad E_n \in (E_{min}, E_{max}) \\ &= \ln\left(E_n - E_{min}\right) && \text{for} \quad E_{max} = +\infty \\ &= E_n && \text{for} \quad E_{max} = +\infty,\ E_{min} = -\infty \end{aligned} \quad , \qquad (24)$$

from (E_{min}, E_{max}) into an infinite range $(-\infty, +\infty)$.

2 CONTROLLING FOTEL OPERATION: INPUT AND OUTPUT FILES

Program FOTEL can run both in DOS and Unix environments. Its operation is basically controlled by various parameters spefified in a file with **an obligatory name FOTEL.IN** (stands for Fotel input). All variables which can become elements of the solution are denoted by running numbers – see below.

Observational data on which the solution is based must be in another input file the name of which is specified by the user in the control file FOTEL.IN . The initial values of the elements of interest can be set either in FOTEL.IN file or in a user-defined input file.

The results of the solution can either be directed to the screen or to two output files specified by the user. The first one contains numerical results of the solution including the errors of derived elements and the other one basically serves for preparation of phase diagrams and contains also the $O-C$ deviations of individual observations from the solution.

The structure and content of input and output files of FOTEL is discussed below:

2.1 The input file with observations

This file has a format which is standard for virtually all reduction and data analysis programs used in Praha and Ondřejov stellar groups. It consists of one record with user-supplied text identification of the dataset, followed by a matrix with columns containing time (usually HJD), dependent variable, weight, dataset number and a numerically coded type of variable. It has a fixed format shown below.

An example of FOTEL input file with observations

Fixed (2F10.4,1X,F8.3,1X,2I5) format

```
--------------------------------------------------------------------------------
        10        20        30        40        50        60        70        80
....|....|....|....|....|....|....|....|....|....|....|....|....|....|....|....|

 V436 Per UBV photometry and all KOREL RVs  - the new 2002 study
42706.7440    5.519      1.00    13    3
37190.5170    5.536      1.00    14    3
44919.2769    5.5130     1.00    18    3
51447.4364    5.8930     1.50    20    3
37190.5170    5.361      1.00    15    4
51447.3558    5.7450     1.50    21    4
51447.3558    4.9070     1.50    22    5
49744.6420  -38.92       1.00     1    1
49744.6420   31.53       1.00     1    2

....|....|....|....|....|....|....|....|....|....|....|....|....|....|....|....|
        10        20        30        40        50        60        70        80
--------------------------------------------------------------------------------
```

As far as the codes of types of variables are concerned, they are specified in Table 1.

Numerical codes of all conceivable elements of the solution available in the current version of FOTEL are listed in Table 2 where also all internally transformed variables are identified.

Table 1: **Numerical coding of different types of variables used in FOTEL**

Code	Variable
1	... radial velocity of the primary
2	... radial velocity of the secondary
3 – 12	... various photometric passbands
13	... radial velocity of the tertiary

2.2 The optional input file with initial values of elements

If requested by key 2 in the control file (see below), the program reads an input file with one record of a header and with records which contain: the number of each specified element (as defined in Table 2), its initial value and its expected range of uncertainty (error). An example of such a file is shown below. The user has to decide which elements has to be specified. (For instance: It is not necessary to specify, say, the limb-darkening coefficients for a RV solution and so on.)

An example of optional FOTEL input file with initial values of elements

Free format

```
--------------------------------------------------------------------------------
        10        20        30        40        50        60        70        80
....|....|....|....|....|....|....|....|....|....|....|....|....|....|....|....|

    V436 Per: elliptical orbit  SEFONO 1 final
1 25.935950665 0.000001
2 43563.3808 0.01
3 0.402 0.01
4 108. 2.  1.890  0.005
5 97. 1.
6 1.06 0.01
7 0.03 0.001
8 0.05 0.001
9 .999420 0.000015
30 0.340   0.01
31 0.395   0.01
32 0.390   0.01
33 0.400   0.01
40 0.340   0.01
41 0.395   0.01
42 0.390   0.01
43 0.400   0.01
110 0.6 0.05
111 0.6 0.05
112 0.6 0.05
113 0. 0.01
120 0. 0.01
121 0. 0.01
122 0. 0.01
123 0. 0.01

....|....|....|....|....|....|....|....|....|....|....|....|....|....|....|....|
        10        20        30        40        50        60        70        80
--------------------------------------------------------------------------------
```

Table 2: Variables which can be either set or become elements of the solution

No.	E_n	E_{min}	E_{max}
1	P Orbital period [time units used in the input data]	0	$+\infty$
2	t_0 Periastron epoch [same time units as data]		
3	e Eccentricity	0	1
4	ω Periastron longitude [in degrees]		
5	K_1 Amplitude of R.V. [km/s]		
6	$q = m_2/m_1 = K_1/K_2$ the mass ratio		
7	R_1 Relative radius of primary [in units of semimajor axis]	0	$1 - E_3$
8	R_2 Relative radius of secondary [in units of semimajor axis]	0	$1 - E_3 - E_7$
9	$\sin i$ sinus of orbital inclination	0	1
10...19	$\delta m_{0,\lambda}$ Amplitudes of periodic light modulation (eq.44) [mag.]		
20...29	A_λ relative extinction coefficients	0	$+\infty$
30...39	$u_{\lambda,1}$ linear darkening coefficient of the primary	0	1
40...49	$u_{\lambda,2}$ linear darkening coefficient of the secondary	0	1
50	$\dot{P}$ Rate of period change [time per time unit]		
51	P' Period of the 3$^{\rm rd}$ star [same time units as data]	0	$+\infty$
52	t'_0 Periastron epoch of the 3$^{\rm rd}$ star		
53	e' Eccentricity of the 3$^{\rm rd}$ star	0	1
54	ω' Periastron longitude of the 3$^{\rm rd}$ star [degrees]		
55	K_{1+2}/K_1 ratio of semi-amplitude of primary to that of the binary motion around the third body		
56	$\dot{\omega}$ Rate of apsidal advance [degrees per unit of time]		
57, 58	$\kappa_{1,2}$ Polytrop. index of 1$^{\rm st}$ and 2$^{\rm nd}$ star	1	$+\infty$
59	Q_1 Synchroniz. index of 1$^{\rm st}$ star		
60...69	$k_{\lambda,1}$ Albedo of the primary		
70...79	$k_{\lambda,2}$ Albedo of the secondary		
80...89	dm_λ/dt Linear trend [mag. per unit of time]		
90	Q_2 Synchroniz. index of 2$^{\rm nd}$ star.		
91...94	T_1, T_2, T_3, T_0 Temperatures [K]		
95	R_3 Radius of the third light		
96...98	$\Delta m_{1,2,3}$ Corrections of magnitudes		
99	D Reddening for reference wavelength (e.g. $V - V_0 = A_{\rm V} \cdot D$)	0	$+\infty$
100...109	λ Wavelengths [řA]		
110...119	$m_{\lambda,1}$ Colour magnitudes of the 1$^{\rm st}$ star [mag.]		
120...129	$m_{\lambda,2}$ Colour magnitudes of the 2$^{\rm nd}$ star [mag.]		
130...139	$m_{\lambda,3}$ Colour magnitudes of the 3$^{\rm rd}$ star [mag.]		
140	dK_1/dt Rate of a linear change of the semi-amplitude of the primary [km/s per unit of time]		
141	$dK_2/dt - K_2/K_1 * dK_1/dt$		
142	$\frac{1}{3}(P_0\ddot{P}_0 - 2\dot{P}_0^2)$		
143...145	$\theta_{Q,1}, \phi_{Q,1}, \dot{\phi}_{Q,1}$ orientation of the 1$^{\rm st}$ star rotation [radian]		
146...148	$\theta_{Q,2}, \phi_{Q,2}, \dot{\phi}_{Q,2}$ orientation of the 2$^{\rm nd}$ star rotation [radian]		
150...159	gravity darkening of the 1$^{\rm st}$ star		
160...169	gravity darkening of the 2$^{\rm nd}$ star		
170...179	correlated darkening of the 1$^{\rm st}$ star		
180...189	correlated darkening of the 2$^{\rm nd}$ star		
190	K_3 Semi-amplitude of the RV curve of the third body [km/s]		

2.3 The control input file FOTEL.IN

As already mentioned, FOTEL.IN control file defines all operations to be carried out by the program. As example of its structure is shown below and a detailed description follows.

An example of FOTEL.IN control input file

Fixed 16I5 format for keys, free format for elements

```
--------------------------------------------------------------------------------
         10        20        30        40        50        60        70        80
....|....|....|....|....|....|....|....|....|....|....|....|....|....|....|....|

    5    1    2    1    1    0    1    0
a            [file with initial parameters]
a.res        [output file with results]
aa           [input file with observations]
a.fot        [output file for Figures]
1 0 0 0 0 0
2 1 0 0 0 0
3 1 0 0 0 0
4 1 0 0 0 0
5 1 0 0 0 0
6 1 0 0 0 0
7 1 0 0 0 0
8 1 0 0 0 0
9 0 0 0 0 0
51 0 0 0 0 0
52 0 0 0 0 0
53 0 0 0 0 0
54 0 0 0 0 0
55 0 0 0 0 0
56 0 0 0 0 0
110 1 0 0 0 0
111 1 0 0 0 0
112 1 0 0 0 0
130 0 0 0 0 0
131 0 0 0 0 0
132 0 0 0 0 0
0 0 0 0 0 0
1 1 0.4
2 2 0.4
13 13 0.867 [redefinition of relative weight of dataset 13]
14 14 0.366
15 15 0.306
16 16 0.193
29 17 1.346 [renaming dataset 29 to dataset 17]
0 0 0
0 0 0
10000        [maximum number of iterations]
0
    5   -1    0    0    0    1

....|....|....|....|....|....|....|....|....|....|....|....|....|....|....|....|
         10        20        30        40        50        60        70        80
--------------------------------------------------------------------------------
```

The control file FOTEL.IN contains **one record with keys controlling program operation** followed by **an unspecified number of records** referring to elements of the solution. Input of these records is terminated by one record containing **six zeros**. After that, it is possible to enter records which can be used to the redefinition of dataset numbers and their relative weights. This input is terminated by a record with **three zeros**.

Note that if some records with dataset redefiniton are entered, there must be two consecutive records with three zeros. The next record (10000 in the present example) specified the maximum number of iterations after which the convergency is terminated (unless the program does not stop sooner, fulfilling the convergency criteria set by the user).

The functions of (presently) nine numerical keys in the first record of FOTEL.IN file are the following:

- **Key 1** Choice of mode of operation
  - 0 ... interactive mode in which program reads all input data from the computer keyboard. It is usually only used for tests of the program run
  - 5 ... standard automatic mode of operation, instructions for run are read from consecutive records of FOTEL.IN file
- **Key 2 (Inpar)** Parameter input, help and termination of program run
  - 1 – 99 ... requires the input of a file with the initial values of elements. The name of this file (in DOS convention!) must be then specified by the user in a record in FOTEL.IN file, immediately following the first record with keys (filename **a** in the example here)
  - > 99 ... instructs program to switch to HELP mode
  - < 0 ... terminates execution of the program
- **Key 3 (Outpar)** Defines output of results
  - 1 ... All results of solutions will be stored in an output file, the name of which is given in the text record in FOTEL.IN file (filename **a.res** in the example here)
  - 2 ... Same as 1 but the output also contains the final values of all elements listed in Table 2. This is recommendable since this allows user to keep track about all details of the solution. Note that not all important elements are otherwise reproduced in the output, for instance limb-darkening coefficients, albedos etc.
- **Key 4 (Input)** Input of observations
  - > 0 ... asks for input of a file with observations. The name of this file (in DOS convention!) must be then specified by the user in another record in FOTEL.IN file (filename **aa** in the example here)
  - 0 ... program is expected to carry out a new solution for observational data entered previously during the run
  - -1 ... theoretical radial-velocity curve for component 1 is calculated for the given elements as a function of phase and stored in the output file for graphs -cf. Key 5 below; Note that period value has to be set to 1.0 for this option! FOTEL.IN must not contain a filename of input data with observations in this option!
  - -1 ... theoretical radial-velocity curve for component 2 is calculated (see the comments for -1).
  - -3 to -10 ... theoretical light curve in passbands Nos. 3 to 10 is calculated (see the comments for -1).
- **Key 5 (Output)** Output of file with data for graphs
  - 0... no output is created
  - > 0 ... data for graphs are stored in file with the name specified on another record in FOTEL.IN file (filename **a.fot** in the example here) as records containing time (HJD), observed value - minus its zero point, orbital phase and $O-C$ from the solution; for triple systems, it is followed by orbital phase of the long orbit, $O-C$ from closer orbit (suitable to plot vs. phase of the long orbit) and $O-C$ from the wide orbit (to be plotted vs. phase of the closer orbit); see also section OUTPUT FILES

- **Key 6 (Echo)** Copy of input data
  - 0 ... input data are not reproduced
  - 1 ... data entered from keyboard are reproduced on the screen
  - > 1... also data from input files are reproduced on the screen
- **Key 7** Quadratic ephemeris requested
  - 0 ... a general formula for a secular change of the orbital period is used in case the perido change is among the elements of the solution (see Hadrava's User's Guide for details)
  - 1 ... the law of period change which strictly corresponds to a quadratic ephemeris in epoch is used instead
- **Key 8** Sets a test of convergency
  - 0 ... the task is performed until the maximum number of iterations specified at the bottom of FOTEL.IN file is achieved
  - $N > 0$ ... the task is executed until a relative accuracy of N valid digits in all elements of the solution is achieved
- **Key 9** Triaxial ellipsoid approximation
  - 0 ... not used

2.4 Main FOTEL output file with results

A beginner might be a bit confused by the relatively complicated structure of the FOTEL output with the results of solutions. For that reason, some explanatory comments are included (in bold [and brown in the colour version]) to the sample output file reproduced below. They should help the reader to understand the FOTEL output.

Here are some more important facts worth mentioning:

1. To identify the input sources, FOTEL first reproduces on output the headers of both input files (if both are used), ie. input file with initial values of elements and input file with observations
2. Info about whether full convergency was achieved or not follows. If the true number of iterations equals to the maximum allowed in the bottom of FOTEL.IN input file, and if the last value is not zero (as in the example below, it means that full convergency was not achieved.
3. As already mentioned, FOTEL calculates the rms errors of the elements from the covariance matrix. This also gives the possibility to derive the munual correlation of individual elements. Altough it may appear boring at first sight, the user should always inspect the correlation coefficients printed after each element of the solution. If some of the coefficients is large, say larger than 0.7, it means that the **individual** values of the respective two elements are not well constrained in the solution and should be adopted with some caution.
4. The statistictics which folows informs the user about relative contributions of individual datasets to the total sum of squares of $O-C$ deviations, about their zero points and rms errors. The latter is particularly useful since it allows the user to access the relative quality of each dataset and perhaps to repeat the solution with weights taking the data quality into account.
5. Note that relative radii are given for spherical as well as triaxial ellipsoid representation.

An example of the main output file with results

```
--------------------------------------------------------------------------------
        10        20        30        40        50        60        70        80
....|....|....|....|....|....|....|....|....|....|....|....|....|....|....|....|


 *** Program FOTEL 2003 for RV/Light-Curve Solution ***
    (C) P. Hadrava        release September 7, 2003
```

Header of input file with parameters

```
     V436 Per: elliptical orbit  SEFONO 1 final
```

Header of input file with observations

```
  V436 Per UBV fotometrie  new 2002 study
```

maximum and true number of iterations and maximum relative error of calculated elements after the last iteration

```
  iter= 7 7  0.000110455058
```

Final values of the elements of solution, their rms errors and the coefficients of correlation of each element in question with all previous elements of the solution

```
  1  El(  1)=0.2593595E+02 +/-0.27758E-05  period
  2  El(  2)=0.4356340E+05 +/-0.52752E-02  epoch
  2  1 El/ZP(  1)  Correl.=-.95756E-01
  3  El(  3)=0.3973566E+00 +/-0.13639E-02  eccentricity
  3  1 El/ZP(  1)  Correl.=-.10206E-01
  3  2 El/ZP(  2)  Correl.=-.98492E+00
  4  El(  4)=0.1895509E+01 +/-0.16452E-02  omega
In degrees:
  4  El(  4)=0.1086046E+03 +/-0.94264E-01
  4  1 El/ZP(  1)  Correl.=0.88706E-03
  4  2 El/ZP(  2)  Correl.=0.99328E+00
  4  3 El/ZP(  3)  Correl.=-.98655E+00
  5  El(  7)=0.3547356E-01 +/-0.84854E-03  radius r1
  5  1 El/ZP(  1)  Correl.=0.14231E+00
  5  2 El/ZP(  2)  Correl.=-.24254E+00
  5  3 El/ZP(  3)  Correl.=0.15201E+00
  5  4 El/ZP(  4)  Correl.=-.24507E+00
  6  El(  8)=0.4556231E-01 +/-0.62065E-03  radius r2
  6  1 El/ZP(  1)  Correl.=-.27102E+00
  6  2 El/ZP(  2)  Correl.=-.62040E+00
  6  3 El/ZP(  3)  Correl.=0.68458E+00
  6  4 El/ZP(  4)  Correl.=-.63537E+00
  6  5 El/ZP(  7)  Correl.=-.53606E+00
  7  El(  9)=0.9994364E+00 +/-0.16296E-04  sin i
  7  1 El/ZP(  1)  Correl.=-.72157E-01
  7  2 El/ZP(  2)  Correl.=0.46702E+00
  7  3 El/ZP(  3)  Correl.=-.38270E+00
  7  4 El/ZP(  4)  Correl.=0.47652E+00
  7  5 El/ZP(  7)  Correl.=-.95593E+00
  7  6 El/ZP(  8)  Correl.=0.28683E+00
  8  El(110)=0.5589233E+00 +/-0.55045E-01  colour mag.1
  8  1 El/ZP(  1)  Correl.=-.18283E+00
```

```
 8   2 El/ZP(  2)  Correl.=-.16755E+00
 8   3 El/ZP(  3)  Correl.=0.25429E+00
 8   4 El/ZP(  4)  Correl.=-.16923E+00
 8   5 El/ZP(  7)  Correl.=-.90460E+00
 8   6 El/ZP(  8)  Correl.=0.83766E+00
 8   7 El/ZP(  9)  Correl.=0.75187E+00
 9   El(111)=0.5334594E+00 +/-0.54936E-01  colour mag.1
 9   1 El/ZP(  1)  Correl.=-.18762E+00
 9   2 El/ZP(  2)  Correl.=-.16886E+00
 9   3 El/ZP(  3)  Correl.=0.25734E+00
 9   4 El/ZP(  4)  Correl.=-.17177E+00
 9   5 El/ZP(  7)  Correl.=-.90345E+00
 9   6 El/ZP(  8)  Correl.=0.83634E+00
 9   7 El/ZP(  9)  Correl.=0.75278E+00
 9   8 El/ZP(110)  Correl.=0.99351E+00
10   El(112)=0.5561502E+00 +/-0.54679E-01  colour mag.1
10   1 El/ZP(  1)  Correl.=-.18840E+00
10   2 El/ZP(  2)  Correl.=-.16993E+00
10   3 El/ZP(  3)  Correl.=0.25822E+00
10   4 El/ZP(  4)  Correl.=-.17278E+00
10   5 El/ZP(  7)  Correl.=-.89977E+00
10   6 El/ZP(  8)  Correl.=0.83409E+00
10   7 El/ZP(  9)  Correl.=0.75028E+00
10   8 El/ZP(110)  Correl.=0.98982E+00
10   9 El/ZP(111)  Correl.=0.98896E+00
19 Zero Point of Dataset(23))=0.552558317E+01 +/-0.85171E-03
19   1 El/ZP(  1)  Correl.=0.59233E-01
19   2 El/ZP(  2)  Correl.=-.14340E+00
19   3 El/ZP(  3)  Correl.=0.14164E+00
19   4 El/ZP(  4)  Correl.=-.12966E+00
19   5 El/ZP(  7)  Correl.=-.10626E+00
19   6 El/ZP(  8)  Correl.=0.15527E+00
19   7 El/ZP(  9)  Correl.=0.47403E-01
19   8 El/ZP(110)  Correl.=0.18344E+00
19   9 El/ZP(111)  Correl.=0.14815E+00
19  10 El/ZP(112)  Correl.=0.14697E+00
20 Zero Point of Dataset(24))=0.534576934E+01 +/-0.95902E-03
20   1 El/ZP(  1)  Correl.=0.34198E-01
20   2 El/ZP(  2)  Correl.=-.12917E+00
20   3 El/ZP(  3)  Correl.=0.13474E+00
20   4 El/ZP(  4)  Correl.=-.12361E+00
20   5 El/ZP(  7)  Correl.=-.62847E-01
20   6 El/ZP(  8)  Correl.=0.10626E+00
20   7 El/ZP(  9)  Correl.=0.26242E-01
20   8 El/ZP(110)  Correl.=0.98843E-01
20   9 El/ZP(111)  Correl.=0.14053E+00
20  10 El/ZP(112)  Correl.=0.97827E-01
20  19 El/ZP( 23)  Correl.=0.82349E-01
21 Zero Point of Dataset(25))=0.452495655E+01 +/-0.11486E-02
21   1 El/ZP(  1)  Correl.=0.24011E-01
21   2 El/ZP(  2)  Correl.=-.11043E+00
21   3 El/ZP(  3)  Correl.=0.11547E+00
21   4 El/ZP(  4)  Correl.=-.10562E+00
21   5 El/ZP(  7)  Correl.=-.49791E-01
21   6 El/ZP(  8)  Correl.=0.88650E-01
```

```
21  7 El/ZP(  9)  Correl.=0.23008E-01
21  8 El/ZP(110)  Correl.=0.79874E-01
21  9 El/ZP(111)  Correl.=0.79808E-01
21 10 El/ZP(112)  Correl.=0.14665E+00
21 19 El/ZP( 23)  Correl.=0.62787E-01
21 20 El/ZP( 24)  Correl.=0.52728E-01
22 Zero Point of Dataset(26))=0.552519049E+01 +/-0.20337E-02
22  1 El/ZP(  1)  Correl.=0.13975E-03
22  2 El/ZP(  2)  Correl.=-.54244E-04
22  3 El/ZP(  3)  Correl.=0.29917E-04
22  4 El/ZP(  4)  Correl.=-.46973E-04
22  5 El/ZP(  7)  Correl.=0.30219E-03
22  6 El/ZP(  8)  Correl.=-.23569E-03
22  7 El/ZP(  9)  Correl.=-.26339E-03
22  8 El/ZP(110)  Correl.=-.30531E-03
22  9 El/ZP(111)  Correl.=-.30251E-03
22 10 El/ZP(112)  Correl.=-.30124E-03
22 19 El/ZP( 23)  Correl.=-.27851E-04
22 20 El/ZP( 24)  Correl.=-.46564E-06
22 21 El/ZP( 25)  Correl.=0.35818E-06
23 Zero Point of Dataset(27))=0.534607949E+01 +/-0.18448E-02
23  1 El/ZP(  1)  Correl.=0.15027E-03
23  2 El/ZP(  2)  Correl.=-.64425E-04
23  3 El/ZP(  3)  Correl.=0.38509E-04
23  4 El/ZP(  4)  Correl.=-.56435E-04
23  5 El/ZP(  7)  Correl.=0.32140E-03
23  6 El/ZP(  8)  Correl.=-.24565E-03
23  7 El/ZP(  9)  Correl.=-.28193E-03
23  8 El/ZP(110)  Correl.=-.32136E-03
23  9 El/ZP(111)  Correl.=-.32011E-03
23 10 El/ZP(112)  Correl.=-.31775E-03
23 19 El/ZP( 23)  Correl.=-.23458E-04
23 20 El/ZP( 24)  Correl.=-.47925E-05
23 21 El/ZP( 25)  Correl.=0.15482E-05
23 22 El/ZP( 26)  Correl.=0.13175E-06
24 Zero Point of Dataset(28))=0.452863495E+01 +/-0.28721E-02
24  1 El/ZP(  1)  Correl.=0.10768E-03
24  2 El/ZP(  2)  Correl.=-.21006E-04
24  3 El/ZP(  3)  Correl.=0.23595E-05
24  4 El/ZP(  4)  Correl.=-.15265E-04
24  5 El/ZP(  7)  Correl.=0.22534E-03
24  6 El/ZP(  8)  Correl.=-.19302E-03
24  7 El/ZP(  9)  Correl.=-.19109E-03
24  8 El/ZP(110)  Correl.=-.23596E-03
24  9 El/ZP(111)  Correl.=-.23429E-03
24 10 El/ZP(112)  Correl.=-.23467E-03
24 19 El/ZP( 23)  Correl.=-.21073E-04
24 20 El/ZP( 24)  Correl.=-.26549E-05
24 21 El/ZP( 25)  Correl.=-.77709E-05
24 22 El/ZP( 26)  Correl.=0.92385E-07
24 23 El/ZP( 27)  Correl.=0.98482E-07
25 Zero Point of Dataset(29))=0.552977023E+01 +/-0.60000E-03
25  1 El/ZP(  1)  Correl.=0.51023E-01
25  2 El/ZP(  2)  Correl.=-.14761E-01
25  3 El/ZP(  3)  Correl.=0.11169E-01
```

```
25   4 El/ZP(  4)  Correl.=-.10621E-01
25   5 El/ZP(  7)  Correl.=0.19725E-01
25   6 El/ZP(  8)  Correl.=-.20387E-01
25   7 El/ZP(  9)  Correl.=-.19608E-01
25   8 El/ZP(110)  Correl.=-.14418E-01
25   9 El/ZP(111)  Correl.=-.20090E-01
25  10 El/ZP(112)  Correl.=-.20424E-01
25  19 El/ZP( 23)  Correl.=0.43727E-01
25  20 El/ZP( 24)  Correl.=0.81596E-02
25  21 El/ZP( 25)  Correl.=0.50669E-02
25  22 El/ZP( 26)  Correl.=0.13960E-04
25  23 El/ZP( 27)  Correl.=0.15810E-04
25  24 El/ZP( 28)  Correl.=0.10912E-04
```

Final values of all elements listed in Table 2; this part of output appears only when Key 3 > 1

```
  1 0.2593595E+02 0.4356340E+05 0.3973566E+00 0.1895509E+01 0.9700000E+02
  6 0.1060000E+01 0.3547356E-01 0.4556231E-01 0.9994364E+00 0.5000000E+00
 11 0.5000000E+00 0.5000000E+00 0.5000000E+00 0.5000000E+00 0.5000000E+00
 16 0.5000000E+00 0.5000000E+00 0.5000000E+00 0.5000000E+00 0.0000000E+00
 21 0.0000000E+00 0.0000000E+00 0.0000000E+00 0.0000000E+00 0.0000000E+00
 26 0.0000000E+00 0.0000000E+00 0.0000000E+00 0.0000000E+00 0.3400000E+00
 31 0.3950000E+00 0.3900000E+00 0.4000000E+00 0.0000000E+00 0.0000000E+00
 36 0.0000000E+00 0.0000000E+00 0.0000000E+00 0.0000000E+00 0.3400000E+00
 41 0.3950000E+00 0.3900000E+00 0.4000000E+00 0.0000000E+00 0.0000000E+00
 46 0.0000000E+00 0.0000000E+00 0.0000000E+00 0.0000000E+00 0.0000000E+00
 51 0.0000000E+00 0.0000000E+00 0.0000000E+00 0.0000000E+00 0.0000000E+00
 56 0.0000000E+00 0.1250000E+01 0.1250000E+01 0.1000000E+01 0.0000000E+00
 61 0.0000000E+00 0.0000000E+00 0.0000000E+00 0.0000000E+00 0.0000000E+00
 66 0.0000000E+00 0.0000000E+00 0.0000000E+00 0.0000000E+00 0.0000000E+00
 71 0.0000000E+00 0.0000000E+00 0.0000000E+00 0.0000000E+00 0.0000000E+00
 76 0.0000000E+00 0.0000000E+00 0.0000000E+00 0.0000000E+00 0.0000000E+00
 81 0.0000000E+00 0.0000000E+00 0.0000000E+00 0.0000000E+00 0.0000000E+00
 86 0.0000000E+00 0.0000000E+00 0.0000000E+00 0.0000000E+00 0.1000000E+01
 91 0.0000000E+00 0.0000000E+00 0.0000000E+00 0.1000000E+05 0.0000000E+00
 96 0.0000000E+00 0.0000000E+00 0.0000000E+00 0.0000000E+00 0.0000000E+00
101 0.0000000E+00 0.0000000E+00 0.0000000E+00 0.0000000E+00 0.0000000E+00
106 0.0000000E+00 0.0000000E+00 0.0000000E+00 0.0000000E+00 0.5589233E+00
111 0.5334594E+00 0.5561502E+00 0.0000000E+00 0.0000000E+00 0.0000000E+00
116 0.0000000E+00 0.0000000E+00 0.0000000E+00 0.0000000E+00 0.0000000E+00
121 0.0000000E+00 0.0000000E+00 0.0000000E+00 0.0000000E+00 0.0000000E+00
126 0.0000000E+00 0.0000000E+00 0.0000000E+00 0.0000000E+00 0.0000000E+00
131 0.0000000E+00 0.0000000E+00 0.0000000E+00 0.0000000E+00 0.0000000E+00
136 0.0000000E+00 0.0000000E+00 0.0000000E+00 0.0000000E+00 0.0000000E+00
141 0.0000000E+00 0.0000000E+00 0.0000000E+00 0.0000000E+00 0.0000000E+00
146 0.0000000E+00 0.0000000E+00 0.0000000E+00 0.0000000E+00 0.0000000E+00
151 0.0000000E+00 0.0000000E+00 0.0000000E+00 0.0000000E+00 0.0000000E+00
156 0.0000000E+00 0.0000000E+00 0.0000000E+00 0.0000000E+00 0.0000000E+00
161 0.0000000E+00 0.0000000E+00 0.0000000E+00 0.0000000E+00 0.0000000E+00
166 0.0000000E+00 0.0000000E+00 0.0000000E+00 0.0000000E+00 0.0000000E+00
171 0.0000000E+00 0.0000000E+00 0.0000000E+00 0.0000000E+00 0.0000000E+00
176 0.0000000E+00 0.0000000E+00 0.0000000E+00 0.0000000E+00 0.0000000E+00
181 0.0000000E+00 0.0000000E+00 0.0000000E+00 0.0000000E+00 0.0000000E+00
186 0.0000000E+00 0.0000000E+00 0.0000000E+00 0.0000000E+00 0.0000000E+00
```

The following statistics informs about relative contributions of individual datasets to the total sum of squares of $O-C$ deviations after the solution, zero levels of individual datasets, their rms errors and global weights assigned to each dataset

```
*** NUMBER OF POINTS=2248 TOTAL SUM =0.248858E+00 ***
   5    1    2    3    4    1    1    0    0    0
VARIABLE  3, IN DATASET 1314, MIN. VALUE=0.548600E+01 MAX. VALUE=0.596500E+01
VARIABLE  4, IN DATASET 1521, MIN. VALUE=0.530900E+01 MAX. VALUE=0.579300E+01
VARIABLE  5, IN DATASET 1622, MIN. VALUE=0.445000E+01 MAX. VALUE=0.497500E+01
Dataset  Var.type  Sum contrib.    Zero level        rms  WEIGHT PER UNIT
   23        3     0.5378E-02        5.5256        0.0066  0.1000E+01
   24        4     0.6941E-02        5.3458        0.0075  0.1000E+01
   25        5     0.9322E-02        4.5250        0.0087  0.1000E+01
   26        3     0.1005E-03        5.5252        0.0047  0.1000E+01
   27        4     0.8270E-04        5.3461        0.0043  0.1000E+01
   28        5     0.2004E-03        4.5286        0.0067  0.1000E+01
   29        3     0.3927E-02        5.5298        0.0061  0.1000E+01
photometry         0.2489E+00                      0.0102
radial vel.        0.0000E+00                      0.0000
```

Principal results of the solution are summarized in a condensed form below

```
! PERIOD            =    25.935947974
! PERIASTRON EPOCH = 43563.397140693
! ECCENTRICITY      =     0.397356584
! PERIASTRON LONG. =   108.604648067
! K1                =    97.000000000
! K2                =    91.509433962
```

Phases relative to the periastron epoch given above as well as epochs of all important orientations of the system in orbit are given

```
Phase of RV max =-0.172719713 T= 43558.917491196
Phase of RV min = 0.091523278 T= 43565.770883659
Phase of conj.1 =-0.020659049 T= 43562.861328660 0.0205355
Phase of conj.2 = 0.392494878 T= 43573.576867428 0.0453459
Phase of min. 1 =-0.020652309 T= 43562.861503490 0.0205354 0.0001039
Phase of min. 2 = 0.392422230 T= 43573.574983223 0.0453449-0.0002296
      Fi(m),   mi sin3i,   mi [Msun]; i=1,2:
 0.18954E+01 0.67532E+01 0.67646E+01
 0.15914E+01 0.71584E+01 0.71705E+01
a1,2,0*sin i= 0.21221E+00AU= 0.45623E+02Rs,   a= 0.21233E+00AU= 0.45649E+02Rs
a1,2,0*sin i= 0.20020E+00AU= 0.43041E+02Rs,   a= 0.20031E+00AU= 0.43065E+02Rs
a1,2,0*sin i= 0.41241E+00AU= 0.88664E+02Rs,   a= 0.41264E+00AU= 0.88714E+02Rs
Inclination=88.076 sin i =0.999436
1st star, periastron and apastron radii and periastron Roche lobe
 RADIUS = 0.035473563 A= 0.035481779 B= 0.035470273 C= 0.035468642
 RADIUS = 0.035473563 A= 0.035474722 B= 0.035473799 C= 0.035472168
 RADIUS = 0.235581182 A= 0.312002982 B= 0.230194945 C= 0.227466419
2nd star, periastron and apastron radii and periastron Roche lobe
 RADIUS = 0.045562308 A= 0.045582295 B= 0.045554417 C= 0.045550232
 RADIUS = 0.045562308 A= 0.045565194 B= 0.045562959 C= 0.045558771
 RADIUS = 0.242273793 A= 0.319537634 B= 0.236988776 C= 0.234093186
VARIABLE= 3 M1-M= 1.0676 M2-M= 0.5087 M3-M= 0.0000
VARIABLE= 4 M1-M= 1.0517 M2-M= 0.5183 M3-M= 0.0000
VARIABLE= 5 M1-M= 1.0659 M2-M= 0.5097 M3-M= 0.0000
PHASE DISTRIBUTION FOR PER=0.259359E+02 E=0.435634E+05
 5 10 15 20 25 30 35 40 45 50 55 60 65 70 75 80 85 90 95100
35 39 55 35 29136 65459109118 99 88 31 27 46 87 67 92107524
```

2.5 Output file with data for graphs

Since FOTEL was gradually developed for both, DOS and Unix systems, it is not creating any graphical outputs and this is left on the user to do graphs with her/his own software. To help in this respect, however, FOTEL provides necessary information in the form of output file for graphs. Two examples of such a file are shown below. Note that there is naturally a difference between this output for binaries and for triple systems:

1. Binary system See the first example in Table 3 below: The output file contains the following columns: time (HJD-2400000 or whatever time unit was used in the input data with observations), variable − zero point of the respective dataset, orbital phase, and $O-C$ from the final solution. The reason why the observables are provided with the zero point subtracted is obvious: If - for example - the input data was photometry from several sources, the plot of second column vs. phase will provide the net light curve corrected for possible systematic differences in the zero points of individual data.

2. Triple system See the second example in Table 4 below: The output file contains the following seven columns: time from input file, variable − zero point of the respective dataset, orbital phase for the first, shorter orbit, final $O-C$ from the solution with both orbits solved, orbital phase for the longer orbit, $O-C$ with respect to shorther orbit only, and $O-C$ with respect to long orbit only. This means that if the user wants to see the final RV curve of the closer, shorter-period system, prewhitened for the longer orbit, he/she will simply plot column 3 vs. column 7. On the one side, the RV curve of the wide, longer-period orbit results from plotting column 5 vs. column 6.

3 PRACTICAL HINTS HOW TO USE FOTEL

3.1 General comments

Dr. P. Hadrava did his best to include a variety of possibilities, especially for the light-curve solution into FOTEL. As a consequence, the usage of the program is not trivial and one has to be cautious when specifying particular tasks. The user should always keep the following simple facts in mind:

1. The values of certain key elements like orbital period, rate of its change or the rate of apsidal advance cannot be derived, but only improved during the process of solution with FOTEL (or any other similar program).

   The value of the orbital period should be found via period finding techniques (Fourier, Stellingwerf's PDM etc.)

   A secular change of the period and the rate of periastron advance can be estimated from preliminary solutions for data subsets covering suitably chosen shorter time intervals.

   Assume for simplicity two solutions for data subsets referring to two, distant enough mean epochs T_1 and T_2. If – for instance – the values of the orbital period obtained from these solutions, P_1 and P_2, respectively, differ from each other significantly (judging by their respective errors derived by FOTEL), one can estimate the rate of period change simply as

$$\dot{P} \sim \frac{P(T_2) - P(T_1)}{T_2 - T_1}. \tag{25}$$

   A similar estimate of the apsidal advance can also be obtained.

2. Never start your solution for new, unknown data set, for which you only have preliminary, inaccurate estimates of the values of input paramaters, allowing immediately for a free convergency of all desired elements of the solution.

   An example: One can derive unknown value of the longitude of the periastron by simply keeping first some reasonable, non-zero value of the orbital eccentricity, semiamplitude of the RV curve and orbital period all fixed and allowing for a free convergency of only ω and periastron epoch. Starting the trial calculations subsequently with values of ω from all four quadrants, one easily finds the value of ω giving the lowest rms error of the solution. Then allow, step by step, for the convergency of the velocity semiamplitude, then eccentricity and only then of all elements of the solution, adopting the improved value of each respective element at each step.

Table 3: First example of output FOTEL file with data **for graphs**: **a binary system**. This example is based on a light-curve solution for binary system V436 Per

HJD-2400000	$V - V_{\mathrm{max}}$	Phase	$O-C$
42706.7500	-0.00124	0.97066	-0.00159
42706.7710	0.00476	0.97147	-0.00184
42706.7770	0.01076	0.97171	-0.00137
42706.7990	0.04476	0.97255	0.00315
42706.8090	0.06076	0.97294	0.00176
42706.8140	0.07076	0.97313	0.00226
42706.8290	0.09276	0.97371	-0.00720
42706.8340	0.11076	0.97390	-0.00059
42706.8440	0.13676	0.97429	0.00141
42706.8600	0.17276	0.97491	-0.00397
42706.8660	0.19076	0.97514	-0.00196

Table 4: Second example of output FOTEL file with data **for graphs**: **a triple system**. This example is based on an orbital solution of the RV-curve of 55 UMa

HJD-2400000	$RV - \gamma_{\mathrm{vel.}}$	Phase (P_{short})	$O-C_{\mathrm{final}}$	Phase (P_{long})	$O-C_{\mathrm{from\ short}}$	$O-C_{\mathrm{from\ long}}$
34831.7730	-45.45056	0.33339	-2.10020	0.67190	2.61246	-50.16322
34852.7478	-0.35056	0.54640	-5.28096	0.68312	-1.09146	-4.54005
34867.7567	-19.45056	0.42336	5.14361	0.69115	8.94146	-23.24841
34887.7790	-62.65056	0.26340	-4.54043	0.70185	-1.28690	-65.90408
35127.0020	67.74944	0.93485	-0.82470	0.82978	-5.48520	72.40994
35127.0350	63.04944	0.94777	1.09227	0.82980	-3.56939	67.71111
35181.8016	-33.65056	0.39248	7.57520	0.85908	1.01667	-27.09203
35181.8466	-35.65056	0.41010	1.60570	0.85911	-4.95434	-29.09052
35202.7677	7.44944	0.60208	-1.05787	0.87029	-8.30774	14.69932
35202.7887	7.64944	0.61030	-2.98371	0.87030	-10.23426	14.89999
35207.8434	7.44944	0.58954	2.30787	0.87301	-5.10523	14.86255
35216.7909	-66.95056	0.09308	-0.59656	0.87779	-8.29295	-59.25416
35216.8179	-61.85056	0.10365	8.59536	0.87781	0.89812	-54.15332
35229.6809	-87.05056	0.14035	-8.31636	0.88468	-16.40991	-78.95700
35925.7601	43.54944	0.70087	-2.42836	0.25691	-0.13700	41.25809

3. A frequent beginner's mistake is to allow for a convergency of some element which is not constrained by the particular set of input data, for instance convergency of semiamplitude of the RV curve for purely photometric observations. Program then usually crashes, reporting some calculation error, functions outside the allowed range and so forth.

4. A simultaneous solution of RV and light curve deserves special attention. If both types of observations are sufficiently represented in the input data, it is advisable – after trial iteration – to vary the global weights of each of the data type in such a way that each of them affects about one half of the total sum of squares of $O-C$ deviations and has, therefore, a similar effect on the resulting elements.[1]

5. Program FOTEL also allows determination of the mass ratio from photometry as one of the elements. In my experience this is a thing which is better to avoid. First, the functional dependence of the equipotential on the mass ratio is a very slowly varying function which – when inversed – leads to a large error in the photometric mass ratio. Second, the result must also be affected by several simplifications used in FOTEL.

6. In principle, it is also possible to include the linear limb-darkening coefficients into the solution but it is also better to forget this possibility. As one can easily verify, the solution of a light curve is usually quite insensitive to even rather large changes of the limb-darkening coefficients. This in turn means that the values of these coefficients in the solution are poorly constrained and the solution will lead to unrealistic values in most cases.

3.2 Solution of a RV curve only

A solution of a RV curve is usually a relatively simple and easy task. Some caution must only be exercised for eccentric orbits if the longitude of periastron and time of periastron passage are not known in advance. An advice how to proceed has already been given above. Actually, the probable value of ω (element 4) can be guessed from the shape of the observed RV curve. Sometimes, it happens that the convergency is succesfully finished and than the program crashes while writing the results. This is usually caused by the fact that the program always calculates also the dimensions of the Roche lobes around the components and the actual radii (which are not, of course, a part of the RV solution) get over these limits. I usually succeded to get around this problem simply by setting very small relative radii (elements 7 and 8), something like 0.001 in the file with initial values of the elements.

So - for normal binaries, one allows for the convergency of elements 1 (if the data cover a sufficiently long time interval and are reasonably distributed in orbital phase), 2, (3 and 4 for binaries with eccentric orbits), 5 (and 6 if RVs for the secondary are also available in the input data file).

A special care must be exercised if one wants to include also a secular change of the orbital period (element 50) and/or the rate of apsidal motion (element 56) into elements of the solution. Any attempt to find out values of these elements by trial-and-error, simply putting in some arbitrary initial values and executing FOTEL, must in all probability fail an will get you into a false local minimum in the parameter space. The only save way how to start is to proceed as indicated in subsection 3.1 above: derive the inital estimate from a comparison of at least two solutions for local subsets of observations, distant enough from each other in time.

For triple stars, one can, in principle, allow also for convergency of elements 51-55 if tha data are rich enough to allow that.

3.3 Solution of a light curve only

A proper usage of various options which the program offers is a bit tricky.

For the clarity of explanation, let us demonstrate the usage of various options of the program on the particular example of photometric data transformed to the standard Johnson UBV system. Let us further adopt the following coding of the types of input data with observations for the individual passbands:

$$\begin{array}{ll} \text{V} & 3 \\ \text{B} & 4 \\ \text{U} & 5 \end{array} \tag{26}$$

When preparing the job for the light-curve solution, first of all do not forget to set the values of limb-darkening coefficients (elements Nos. 30 to 32 for the V, B and U of the primary in our example, and 40 to 42 for the secondary).

[1] See subsection 2.3 how to do that in practice.

For stars closer to contact configuration, you should also set albedos (elements 60-62 and 70-72 for the primary and secondary, respectively) and the gravity darkening coefficients (elements 150-152 and 160-162 for the primary and secondary, respectively). As you can easily verify, they are all set zero in the program if you do not define them.

The user can decide for one of the two following options when solving a light curve with FOTEL:

1. The first one is to converge the so called colour magnitudes (elements Nos. 110-119 and 130-139 in case of non-zero third light; 110-112 in our example). In that case, one gets zero levels of all photometric passbands corresponding to the respective magnitudes of the whole system at maximum light m. Further, the solution contains also magnitude differences $m_{\rm j}$ between individual binary or triple-star components and the overal magnitude of the system at maximum light

$$m_{\rm j} - m = -2.5 \log \frac{L_j}{L_1 + L_2 + L_3} \tag{27}$$

   for all passbands from the input data.

   Let us assume for example that we derived a simultaneous solution of the light curve observed in standard UBVpassbands, with input channels coded as defined above. Then using the values of maximum levels (zero levels) in channels 3, 4, and 5 gives the respective values of$m$ in equation (27) from which one can therefore obtain the individual UBVmagnitudes of the binary or triple-star components. In case of standard UBV photometry these may provide the user with information about spectral types. To obtain it, one has, of course, to carry out a standard derredening, individually for each component. If everything is all right, one should arrive at the same $E(B-V)$ and $E(U-B)$ for both or all three stars within the limits of expected accuracy of the data. Auxiliary programs BINTEF and TRITEF (see section 4) can help you with this task.

2. The other possibility is to carry out convergency in effective temperatures, i.e. elements Nos. 91, 92 (and 93 if third body is also considered). If one wants to include the third body into the solution, it is also necessary to set element No. 95 to 1 in the input parameters. If the user specifies also the values of interstellar extinction (elements Nos. 20 to 29), it is possible to derive also the interstellar reddening (element No. 99) from the solution. Program assumes blackbody radiation of the components.

   Using the same example as above: In this case the zero points in magnitude channels 3 to 5 will represent dereddened V_0, B_0 and $U-0$ magnitudes at maximum light for the whole system. Note that if one adopts the coefficients of relative interstellar extinction from the latest edition of Allen's tables, $A_{\rm V} = 1.00$, $A_{\rm B} = 1.31$ and $A_U = 1.56$, one has

$$V - V_0 = A_{\rm V} \cdot D = D = 3.1 \cdot E(B-V). \tag{28}$$

   Consequently, using equation (27) again, one obtains directly the dereddened magnitudes of the components of the system. Note that even in this case, each data set has to have its individual dataset number (different for V, B and U in our example).

Let us mention also some more practical advice: If one tries to derive the light-curve solution for a system for which no previous solution is available, it is advisable to make at least some extimate of possible relative radii of both stars. Note that if you have some idea at least about the probable mass of the more massive component of the binary, the third Kepler law gives you a very reasonable estimate of the distance between the centres of the components a since you know that the total mass must be between M_1 and $2M_1$. Then you can assume either, for instance mass-sequence radii or a relative radius of the Roche lobe, depending on with which system you are dealing and what you know about it.

When solving a light curve of a binary, one has to allow for convergency of elements Nos. 1 (if your observations are well distributed in time and orbital phase to allow a meaningful improvement of the orbital period), 2, (3 and 4 if the orbit is eccentric), 7, 8, 9, and 110-119 (depending how many passbands are in the data; only 110-112 for UBV data, for instance). It would not be wise, however, to allow for the convergency of all those elements right at the beginning. The initial trial solutions should only allow convergency of the epoch (element 2), one of the radii (that one for which you have a worse initial estimate; elements 7 and 8) and always of the colour magnitudes 110 up to 119. Caution: You must never allow for simultaneous convergency of elements 110 up to 119 and 120 up to 129 since the colour magnitudes of the two components of any binary are mutually related through the total magnitudes of the binary at maximum light and do not represent mutually independent quantities!

Be prepared that if you execute a light-curve solution with several hundreds or thousands of observations, secured in several passbands, and if you set the run to continue down to machine accuracy, such a task may take several hours. At the same time, when you are sure that you are already heading for the final solution, it is worth to go down to the machine accuracy since the multiparameter space for photometric solutions is often very flat in some parts and you could miss the true minimum when terminating the run too early.

3.4 A combined solution of RV and light curve

It is one of the great advantages of FOTEL that one can derive combined solution of the RV and light curve in one run since this provides the best accuracy of the detemination of various important elements. In an ideal case, one can allow for the simultaneous convergency of all elements mentioned in the previous two subsections. A recommendable procedure how to yet improve the accuracy of the final solution is to run first the solution as it is. Using its result, tune the relative global weights of RVs and photometry in such a way that each of those principal datasets contributes about one half of the sum of squares of $O-C$ deviations. Then run a new, final solution with these global weights.

3.5 A few tips for a more sophisticated use of FOTEL

1. As already mentioned, one can usually achieve what she/he wants having a generous limit of up to 20 elements which can be converged simultaneously. Note that the elements with a linear dependence like the semiamplitude of the primary or the zero levels of all datasets can be calculated directly and do not enter this 20-parameter limit.

   However, studying, for instance, a triple system with its closer pair having measurable apsidal motion and having at the same time photometry from many photometric passbands, one can get into the trouble with the above limit. The solution is to profit from the fact that FOTEL.IN command file allows the user to concatenate the tasks executed by the program. One can, therefore, run some number of iterations solving for the first twenty elements, then to switch to another solution with other twenty (perhaps partly overlapping) elements, starting with the values of the elements obtained in the previous solution, etc. One should, of course, verify the result in such cases via changing the order in which subsets of elements were iterated.

2. Even more is possible: I have myself used this program several times in the past to model a smooth but non-sinusoidal variation of some observed quantity, like central intensity of equivalent width, to get a quantitive mathematical model of the variation, its ephemeris or even to study the residuals frotm the model for the presence of other frequencies.

In conlusion, using FOTEL is not always trivial and one should not run the tasks blindly, but once you master its operation, you will find that it is really a wonderful tool for students of binaries.

4 AUXILIARY PROGRAMS

Auxiliary programs **VOLBA**, All auxiliary programs mentioned here are Fortran77 programs written by Petr Harmanec and can be compiled and executed both under DOS and Unix.

VOLBA2 and **VOLFOT** are available to create FOTEL input file in the obligatory format from arbitrarily free-format matrixes consisting of columns with time and N dependent variables. When executed, all three programs interact with the user and tell him how to proceed. VOLBA2 differs from the simplest version VOLBA in that it also allows the user to add a defined constant algebraically to the variable. This can be used, for instance, when one has the original relative photometry in the form of magnitude differences (variable − comparison) and wants to add the value of the magnitude of the comparison to it. Finally, VOLFOT reads the output data from the Praha/Ondřejov UBV photometric archives and assigns types of variables from 3 to 5 to V, B, and U, respectively.

There is also an auxiliary program **FOTAXUM** which transforms the output of FOTEL with phases from the interval $<0,1>$ to $<-0.25,1.25>$ for better readability of Figures.

Programs **BINTEF** and **TRITEF** calculate the standard deredding for the components of binary and triple systems, respectively, using the results from FOTEL solution of UBV light curves.

5 BUG REPORTS AND NOTES ON IMPLEMENTATION

We did our best to test all the programs on several different types of computers and have reasonable confidence that they can be used under different operating systems. Note, however, that the FORTRAN version of FOTEL is provided in Microsoft Fortran for DOS machines and contains a command

$ LARGE

followed by specification of several larger data fields in the main program. Please, delete this command before compiling the program with any other FORTRAN compiler.

Since the programs can be used under different operating systems and since, for instance the executable Unix versions may not run on different types of computers, we have only included source versions and compiled (executable) versions suitable for personal computers running under the DOS operating system.

Any questions concerning the programs as well as reports of their possible malfunctions should be addressed to the respective author, i.e. to Dr. Petr Hadrava concerning FOTEL and to Petr Harmanec concerning the auxiliary programs.

Any suggestions how to improve this manual will also be appreciated.

Acknowledgements

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