Simulation of free fall and resonances in the forthcoming GOCE mission

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Abstract

GOCE, ESA’s first Earth gravity mission, was launched on 17 March 2009 into a sun-synchronous orbit. Using the full-scale numerical propagator, we investigated the satellite’s free fall from the initial injection altitude of 280 km down to the first measurement phase altitude (at 264 km). During this decay phase the satellite will pass below the 16:1 resonance (268.4 km). The effect of this resonance, together with the uncertainty in the solar activity prediction, has a distinct impact on the evolution of the orbital elements. Then, to maintain a near-constant and extremely low altitude for the measurement operational phases, the satellite will use an ion thruster to compensate for the atmospheric drag. In order to obtain the groundtrack grid dense enough for a proper sampling of the gravitational field, ESA set constraints for a minimum groundtrack repeat period. We studied suitable repeat cycles (resonant orbits) in the vicinity of 16:1 resonance; we found that they differ greatly in stability towards small perturbations of the satellite’s mean altitude and in temporal evolution of the groundtrack coverage. The results obtained from the usual analytical treatment of orbital resonances were refined by more realistic numerical simulations. Finally, we formulated suggestions that might be useful in GOCE orbit planning.

Introduction

ESA’s GOCE satellite

- To date the most advanced gravity space mission
- Successfully launched on 17 March 2009
- The main scientific instrument: gradiometer
- Electric ion propulsion system to counteract drag
- Goal of the mission: to improve the global and regional models of the Earth gravitational field

Orbit of GOCE

- Dusk-dawn nearly sun-synchronous orbit
- Very low circular orbit

Topics of our study

- Free fall in the early orbit phase
- Suitable groundtrack (GT) repeat orbits for the measurement operational phases (MOP)

Free fall of GOCE

- Controlled decay from injection altitude 280 km down to MOP1 altitude 263 km
- Correction of potential launch injection errors
- Check out of gradiometer

Aim of our simulation

- Modelling all the perturbing accelerations
- Reliable prediction of time needed for the satellite to descend to MOP1

Manfest features of graphs

- Atmospheric drag (Fig. 1) ⇒ steady decrease in semimajor axis (SMA) or in mean altitude
- When the satellite is tilted (‘15° ‘tilt’), the drag is increased w.r.t. the ‘nominal’ satellite attitude
- ‘Max’/‘min’ predicted level of solar activity (11-year sunspot cycle) ⇒ increased/decreased atmospheric through strong 16:1 resonance ⇒ quasi-secular changes especially in inclination

Deep solar minimum

- This is the quietest sun we’ve seen in almost a century. (Science@NASA, April 1, 2009)
- A delayed start of the new cycle of solar activity ⇒ MOP1 is planned below the 16:1 resonance

Use of resonant (or repeat) orbits

- Evaluation of lumped geopotential coefficients
- Mission planning for Earth observing satellites (e.g. satellite altimetry)

Analytical treatment of orbital resonances

- Effects of only J\textsubscript{2} perturbation (oblateness)
- Expressions for nodal revolution, 2π/(Ω + M), nodal day, 2π/(ω – Ω), ω, Ω ... Earth angular rate, and secular changes due to J\textsubscript{2}:
  \[ Ω = \frac{3}{2} \frac{ω}{M} \left( \frac{2}{3} \right)^{2} \cos \left( 1 - e^{2} \right) \]
  \[ ω = \frac{3}{2} \frac{ω}{M} \left( \frac{2}{3} \right)^{2} \left( 1 - 5 \cos^{2} \theta \right) \left( 1 - e^{2} \right)^{2} \]
  \[ M = n - \frac{3}{2} \frac{ω}{M} \left( \frac{2}{3} \right)^{2} \left( 1 - 3 \cos^{2} \theta \right) \left( 1 - e^{2} \right)^{2} \]
- Resonance condition for mean motion
  \[ n = \omega + \frac{3}{2} \frac{ω}{M} \left( \frac{2}{3} \right)^{2} \left( 4 \cos^{2} \theta - \frac{3}{2} \cos i - 1 \right) \]
  for given \( ε \geq 0 \) and \( i = R, D, a \)

Resonances and GT coverage

Orbital resonance R:D – the satellite performs R nodal revolutions, while the Earth rotates D times w.r.t. satellite’s precessing orbital plane.

Groundtrack repeat orbit – a GT that repeats after an integer number R of orbital revolutions and an integer number D of nodal days.

Figure 1. Mean orbital elements calculated by NUMINTSAT.

Figure 2. Solar activity cycle.

Figure 3. Histograms of gravitational and nongravitational accelerations acting on GOCE.
Example: CHAMP, GRACE
- Gravity field modelling requires dense enough sampling of Earth surface
- Degradation in accuracy of GRACE monthly gravity solutions near 61.4 (9/04)
- CHAMP passed through 46.3 (10/00), 31.2 (5/02, 10/02, 6/03), 47.3 (11/05, 1/07)

Evolution of groundtrack (GT) coverage
- 977:61 – consecutively filling up the equator during the whole repeat period
- 978:61 – created in two 30-day phases, the half-filled grid being close to 481:30 orbit

Histograms from numerical integrator: 977:61
- We took into account other perturbations (Fig. 3)
- Narrow bars from J2 theory become wider, but still the bars are single-peaked around Δλ (977)
- Altitude 263.9 km from the J2 theory is still valid

Conclusions/suggestions for GOCE
- 16:1 resonance may induce inclination changes ±0.03° ⇒ re-adjust SMA to fulfill the orbit requirements after each pass through resonance
- After adoption of higher value for the drag coefficient, Cd, our prediction of descent times gives comparable results to those of ESA

Study of two suitable 61-day repeat orbits
- 977:61 orbit vs. 978:61 orbit, lower by 4.5 km
- GT grids almost the same with Δλ±41 km
- GT grid of 977:61 laid down consecutively, GT grids almost the same with Δλ±41 km
- Δλ = 0.364 0.366 0.368 0.37 0.372

Current orbital data

Planned mission phases
Comm. @273 km, Apr 2009 – Aug 2009 (5 months)
MOP1 @263 km, Sep 2009 – Mar 2010 (7 months)
HOP1 @273 km, Apr 2010 – Aug 2010 (5 months)
MOP2 @263 km, Sep 2010 – Mar 2011 (7 months)
Planned altitudes lie on the first branch below 16:1 resonance preserves better the repeatability conditions
- The same applies to first branch above 16:1
- Results were tested using both analytical and full numerical orbital propagators

Reference
Bezděk et al., 2009. Simulation of free fall and resonances in the GOCE mission. J. Geodyn., in press, doi: 10.1016/j.jog.2009.01.007

Figure 10. Histogram of Δλ for orbits near 977:61 resonance using the full numerical integrator.

Figure 7. Evolution of GT grids for 977:61 and 978:61 orbits.

Need of SMA adjustment to obtain GT repeat
At first, histograms of node separation for 978:61 orbit produced a double peaked graph – but such a histogram should produce a single peak, maybe spread around the central value 360°/R.

Histograms based on analytical theory: 978:61
- Recall: For GOCE we need no equatorial gaps larger than 42 km (377°)
- After 61-day repeat period, we draw the histograms of lengths of the gaps
- Blue bar – the node separation Δλ=0.368° corresponding to an exact 978:61 repeat
- Altitude reduced by 50 m ⇒ two green bars, etc.
- 150-m height reduction ⇒ histogram in cyan with 0.748° corresponding to 481:30 orbit
- Only a 50-cm disturbance in height shows the instability of 978:61 orbit

Figure 5. GT coverage for different resonant conditions.

GOCE
- A globally uniformly distributed dense coverage ⇒ GT repeat period ≥ two months
- Sun-synchronicity ⇒ inclination 96.7°
- Resonance diagram with 61-day repeat orbits, candidates for MOPs

Figure 6. Orbital resonances predicted for GOCE (i=96.7°).

Equatorial node separation, Δλ
- After the completed repeat period, GT grid should be homogeneous with an equatorial node separation Δλ = 360°/R
- After 61 days, Δλ (977)=0.3685° (41.02 km) and Δλ (978)=0.3681° (40.98 km) ⇒ similar GT grid

Figure 8. Histograms of node separation for orbits near the 978:61 resonance as function of the mean altitude.

Figure 9. Histogram of Δλ for orbits near 977:61.