Calibration of accelerometers aboard GRACE satellites by comparison with GPS-based nongravitational accelerations

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Summary

The proposed calibration method starts with the precise positions of the satellite. The second derivative of the positions yields the total acceleration vector, from which the modelled accelerations of gravitational origin are subtracted. In this way, the nongravitational (NG) acceleration vector is obtained, which is then used as a standard for the calibration of the accelerometer (ACC) data. The main problem of this procedure is the amplification of noise present in the GPS positions, especially at high frequencies. The calibration parameters for the GRACE accelerometers have already been published using other methods. The goal of our study was to obtain not only the calibrated ACC measurements, but also a statistically correct estimate of their accuracy. Thanks to the generalized least squares (GLS) method, such a realistic estimate of the error bars has been acquired. Moreover, in this way the amplification of noise was completely eliminated. The calibration procedure was applied to the ACC measurements covering more than 1.5 years (08/2002–03/2004).

Background and aim

① Forces acting on low Earth satellites

- Satellites at low altitudes: 100–2000 km
- Dominant is the central gravitational field
- Other forces act as small perturbations

② Accelerations of gravitational origin

- Central geopotential term: $\approx 8.5\ \mathrm{m\,s^{-2}}$
- Noncentral geopotential terms, lunisolar perturbations, solid Earth and ocean tides, relativistic effects: $\lessapprox 10^{-2}\ \mathrm{m\,s^{-2}}$

$$|\sum a_{\mathrm{GRAV}}| \lessapprox 8.5\ \mathrm{m\,s^{-2}}$$

③ Nongravitational (NG) accelerations

- Atmospheric drag, radiation pressures (direct solar, albedo, terrestrial infrared)

$$|\sum a_{\mathrm{NG}}| \lessapprox 10^{-9}\text{–}10^{-6}\ \mathrm{m\,s^{-2}}$$

④ Space accelerometers

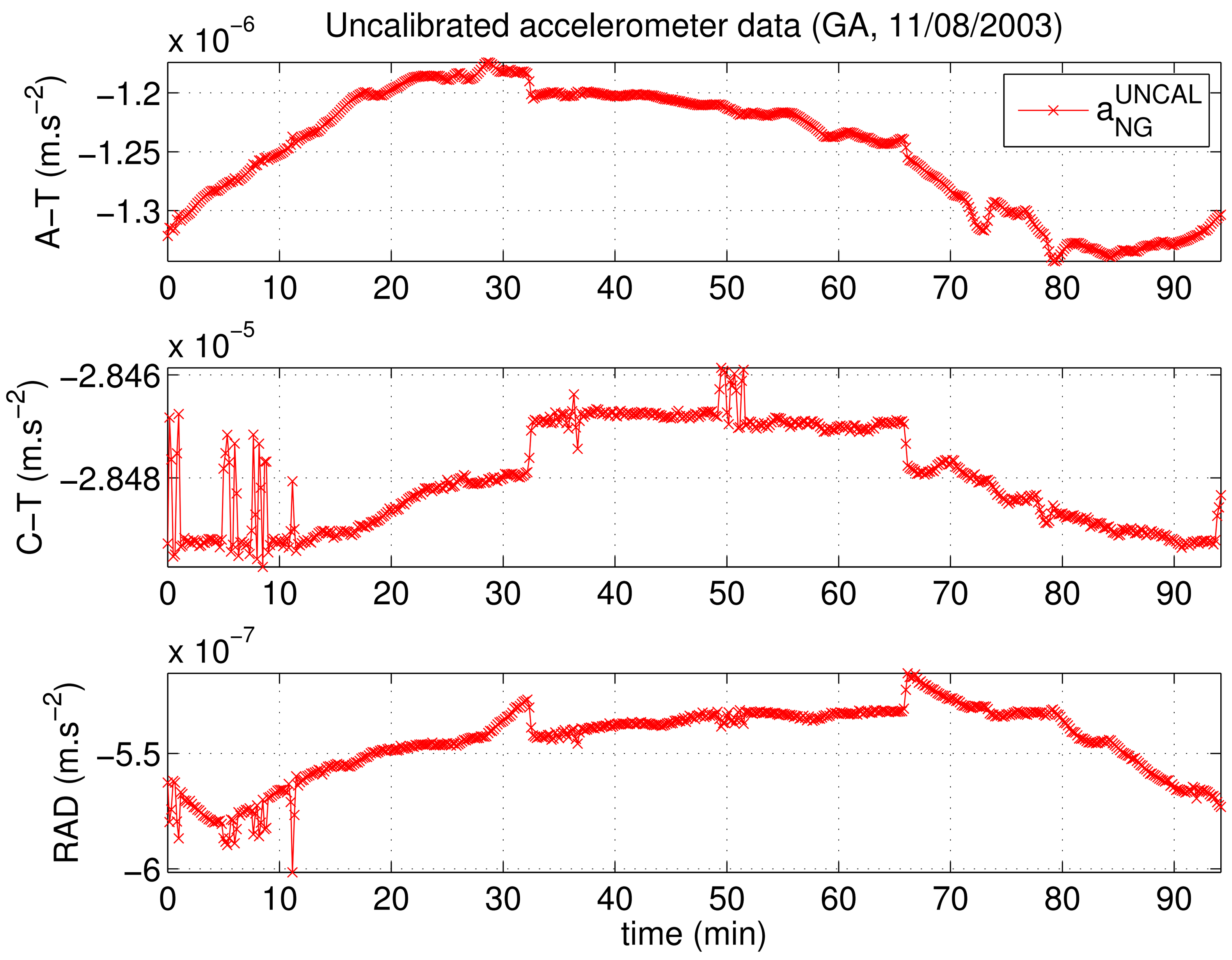
- Designed to measure the tiny NG accelerations
- Actually in space aboard the satellites CHAMP, GRACE, GOCE, planned for SWARM

⑤ GRACE mission

- Launched in 2002 to study the Earth gravity field
- Two satellites in tandem (GA, GB)
- Each satellite carries an accelerometer (ACC)

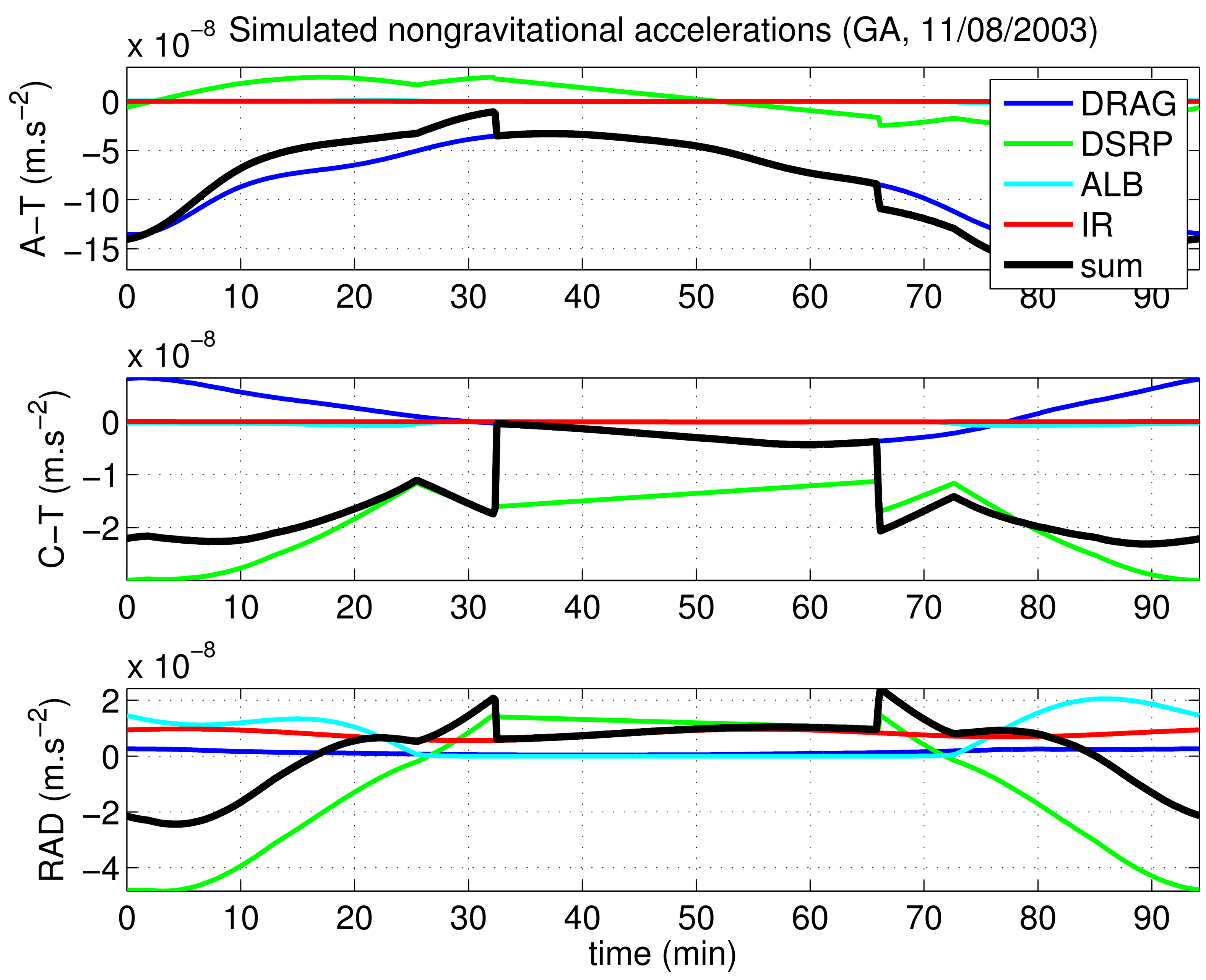
⑥ Uncalibrated ACC data

- Due to the smallness of the NG signal compared to gravity, space accelerometers cannot be calibrated on the ground
- Figure below shows the along-track (A-T), cross-track (C-T) and radial (RAD) components



⑦ Modelled NG accelerations

- On average, the *waveform* of the simulated NG accelerations is quite close to the uncalibrated ACC signal



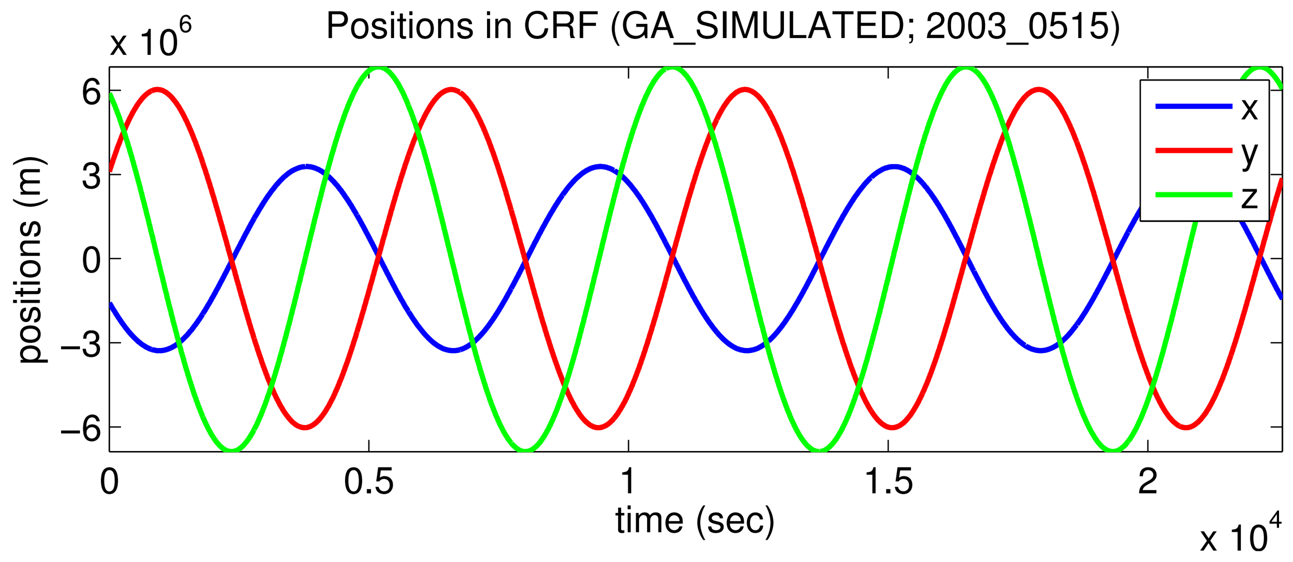
(8) Aim of the study

- To calibrate the ACC measurements and obtain the appropriate uncertainty estimates (error bars)
- Evolution of the calibration parameters over 1.5 yrs of the GRACE mission (08/2002–03/2004)

Method

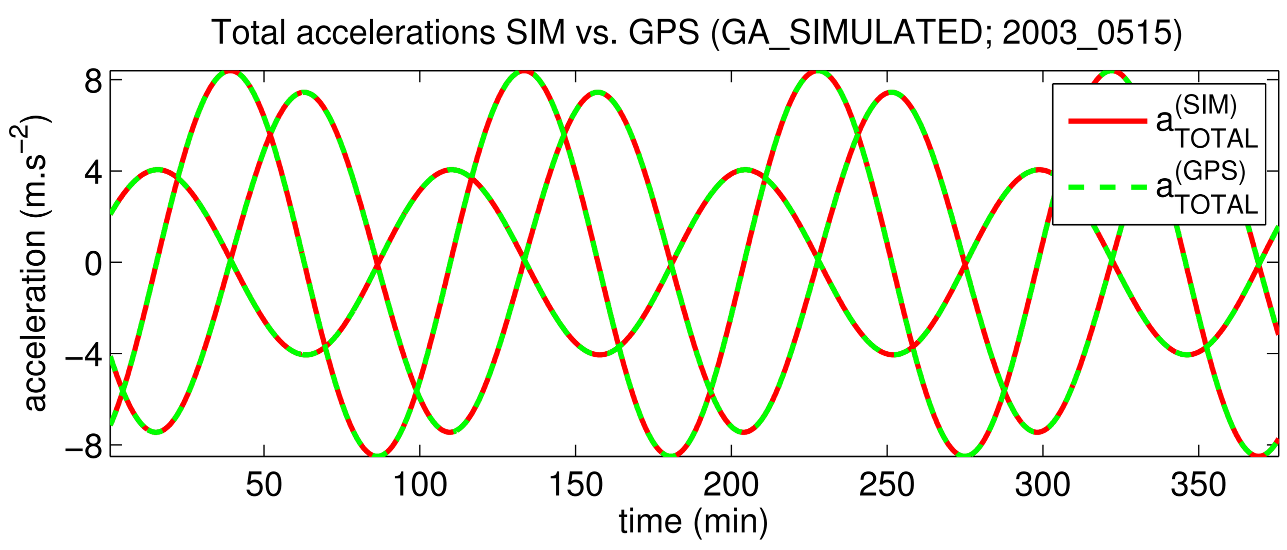
(9) Precise positions

- We start the calibration with the measured positions
- First, a simulated situation: precise positions with an added white noise of variance σ^2=1 cm



(10) Second derivative ⇒ total acceleration

- Second derivative implemented as a linear filter F
- The total accelerations from GPS are virtually the same as the original simulated ones



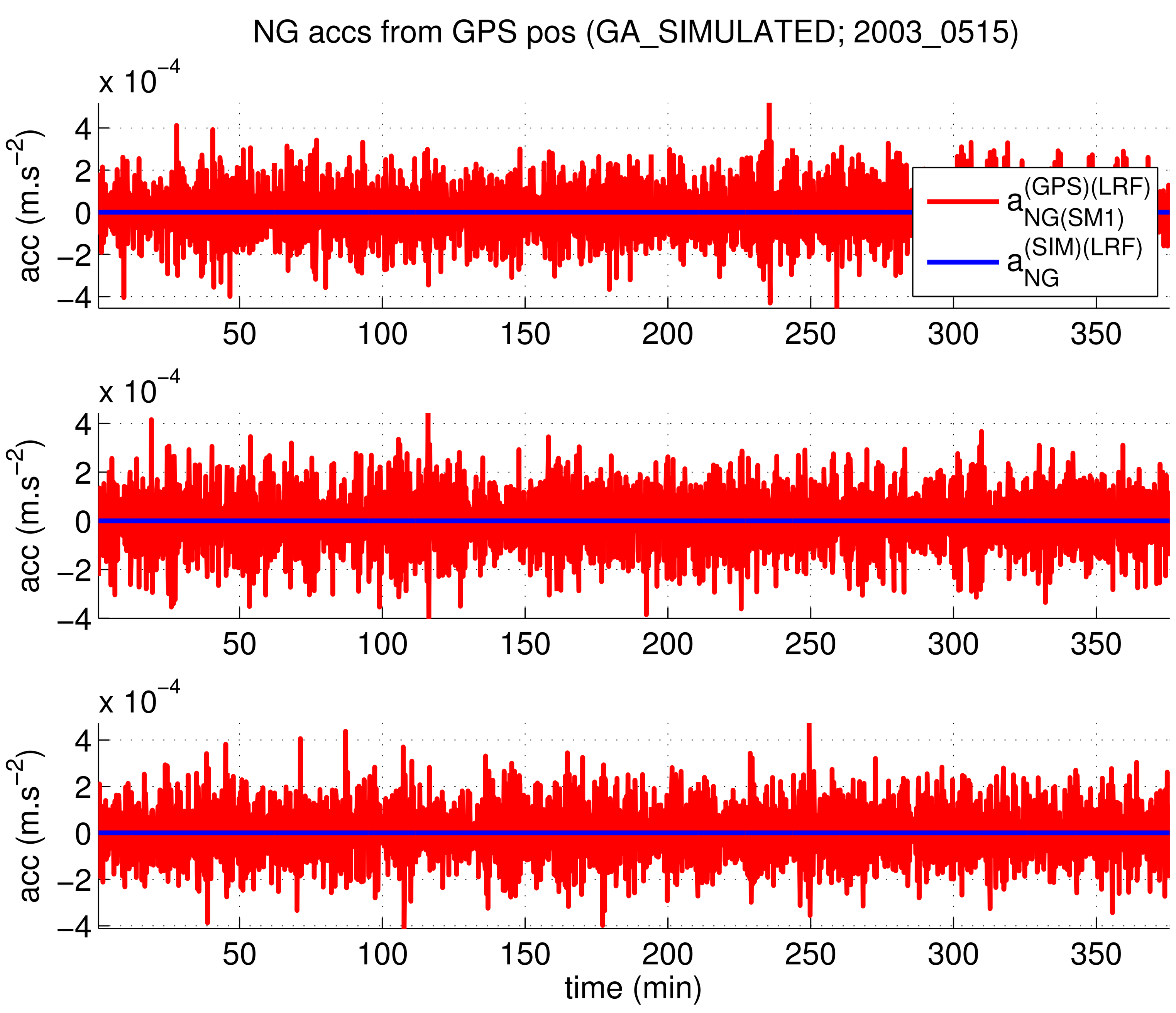
(11) GPS-based NG accelerations

- Are obtained after the subtraction of the modelled accelerations of gravitational origin

$$\boldsymbol{a}^{(\mathrm{GPS})}_{\mathrm{NG}} = \boldsymbol{a}^{(\mathrm{GPS})}_{\mathrm{TOTAL}} - \boldsymbol{a}^{(\mathrm{SIM})}_{\mathrm{GRAV}}$$

(12) Problem of the amplified noise in $\boldsymbol{a}^{(\mathrm{GPS})}_{\mathrm{NG}}$

- The second derivative filter amplifies the noise in positions, especially at high frequencies (HF)
- The "true" signal $\boldsymbol{a}^{(\mathrm{SIM})}_{\mathrm{NG}}$ is buried in noise



(13) Calibration equation

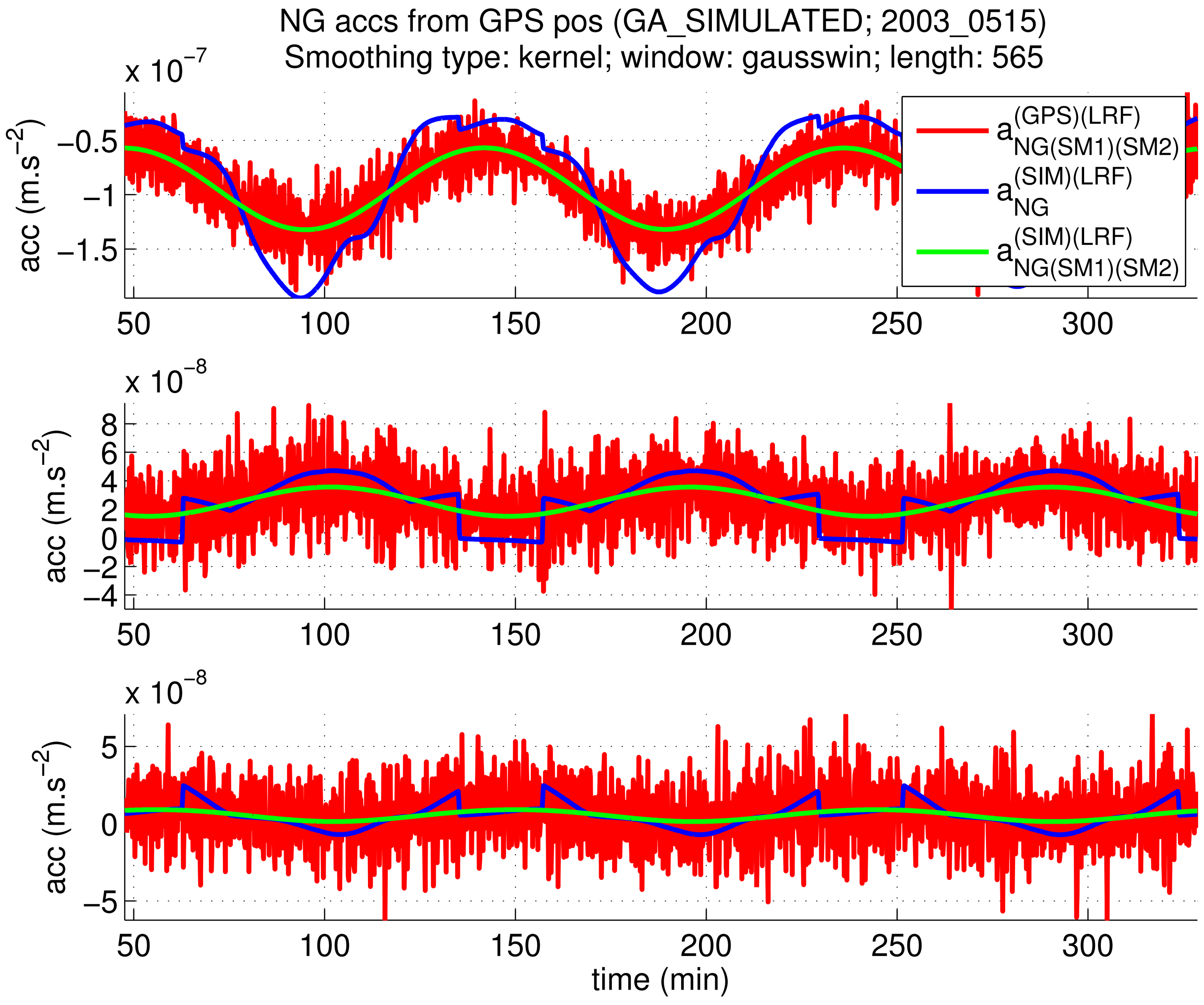
- Bias B and scale factor S to be obtained from

$$\boldsymbol{a}^{(\mathrm{GPS})}_{\mathrm{NG}} = \boldsymbol{B} + \boldsymbol{S} \cdot \boldsymbol{a}^{(\mathrm{UNCAL})}_{\mathrm{NG}}$$

- We have one equation for each ACC axis; in nominal attitude the ACC axes are very close to (A-T, C-T, RAD) directions

(14) Removal of HF noise by smoothing

- First attempt at coping with the HF noise
- Not only the noise, but also the signal is smoothed at HF
- Only low frequency portion of $\boldsymbol{a}_{\mathrm{NG}}$ is recovered
- The smoothing filter increases the correlation of noise
- Because of these drawbacks, we looked for better solution



(15) Problem of autocorrelated noise

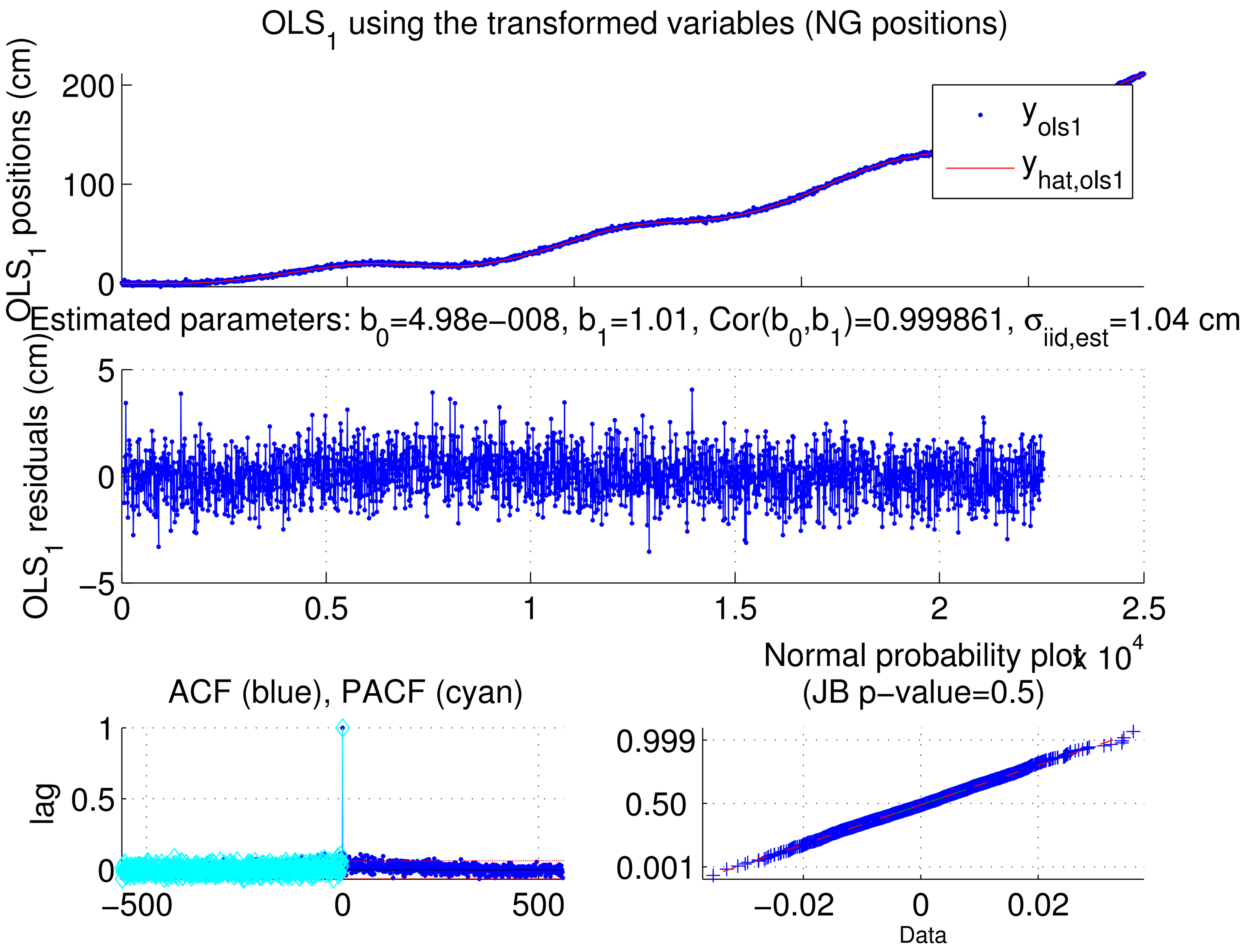
- Ordinary least squares (OLS) provide correct uncertainty estimates for the fitted parameters (usual "3-σ rule"), if the errors are independent and normally distributed
- If the random errors are positively correlated, uncertainty in the fitted parameters is underestimated ⇒ false sense of accuracy
- This applies to calibration equation (13), because filter (10) introduces a correlation structure into the random noise

(16) Generalized least squares (GLS)

- If errors in an OLS problem are correlated, GLS defines a linear transformation W such that the new covariance matrix is diagonal
- In the transformed variables, usual OLS are used to find the regression parameters with correct estimates of their uncertainties
- If C is the covariance matrix of the original OLS problem, possibly with non-zero off-diagonal elements, $C \neq \sigma^2 I$, then $W = T^{-1}$, where $C = TT'$ (T is sometimes called "square root" of C)

(17) Use of GLS to remove autocorrelation

- In fact, non-diagonal covariance matrix was generated by the second derivative filter F: $C = \sigma^2 FF'$, where σ^2 is the variance of the white noise in positions
- Therefore, finding the GLS transformation matrix is straightforward, $W = F^{-1}$. Filter F being of FIR type, the discrete convolution may be formulated using a matrix; we then find W as the inverse of the corresponding filter matrix
- After applying W to eq. (13), the residuals become again uncorrelated and the original σ^2 may be recovered
- In the figure below, only the A-T component is shown; the simulated calibration parameters $B = 5 \times 10^{-8}$ m s^{-2}, S=1 were found correctly



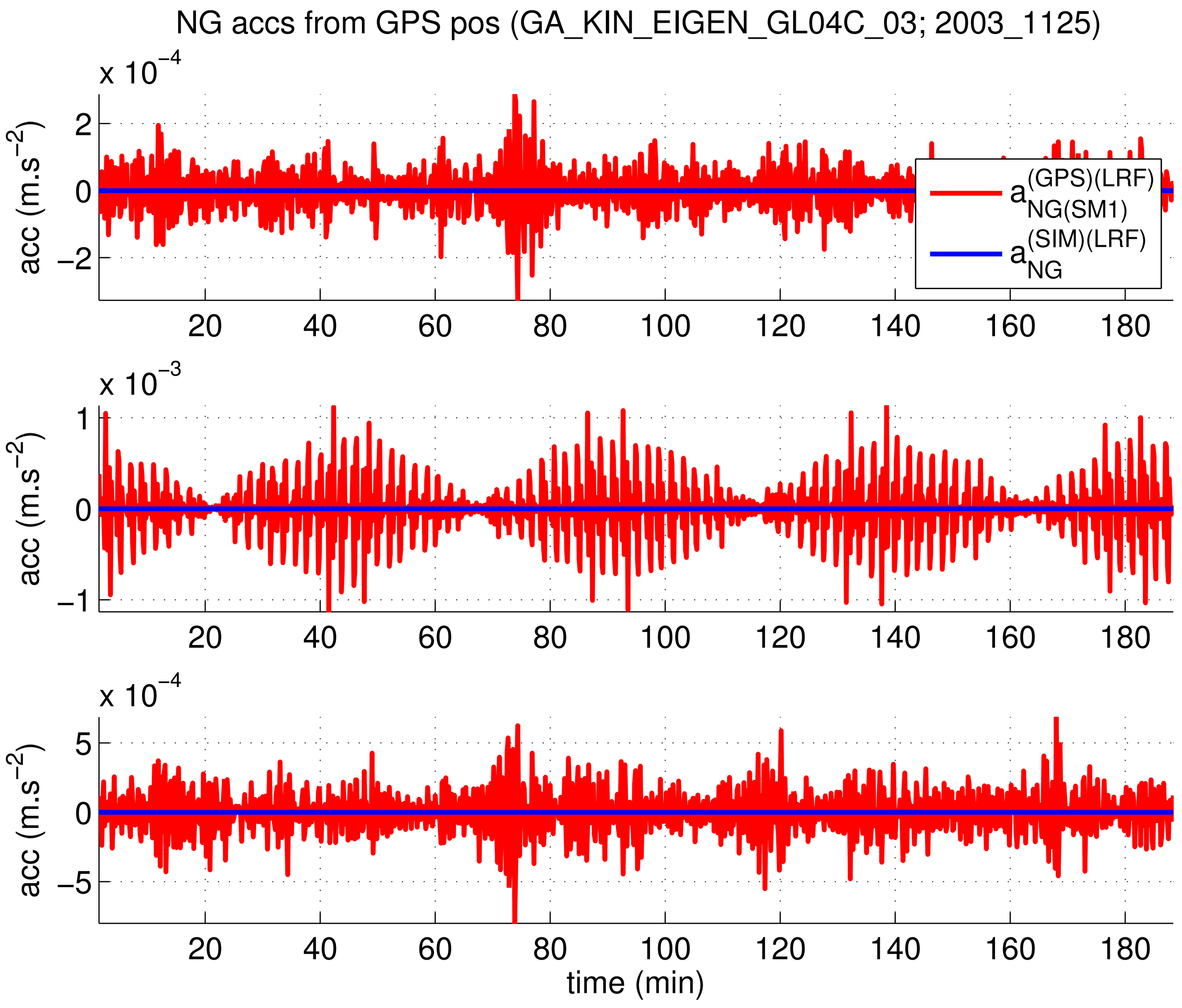
18 Elimination of HF noise in "NG positions"

- As the GLS transformation $W{=}F^{-1}$ is in fact the inverse to the second derivative filter, the "NG positions" are obtained as a sort of double integral of $\boldsymbol{a}_{\mathrm{NG}}^{(\mathrm{GPS})}$
- Effectively, we got back into the positions, but now with the gravitational signal removed
- Here, as an estimate of the error variance, the GLS recovered the original white noise σ^2, defined in 9

Results

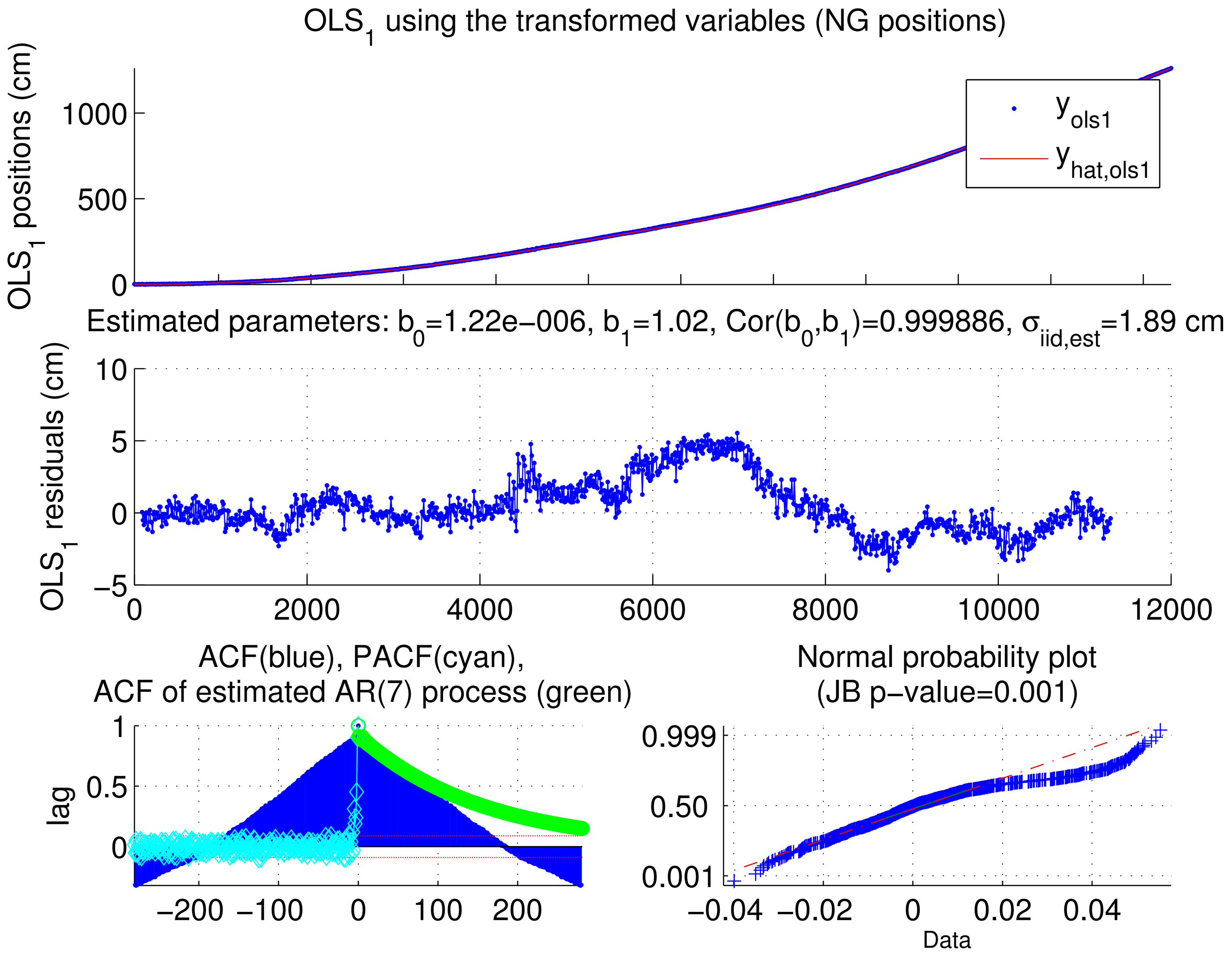
19 Calibration over the period of a few revolutions

- Real-world 10-sec kinematic orbits of GRACE satellites used
- GPS-based NG accelerations are now obtained using 9–11
- The noise in $\boldsymbol{a}_{\mathrm{NG}}^{(\mathrm{GPS})}$ is amplified as in 12
- Note: This seemingly easy step involves a great deal of work on precise numerical orbit propagator



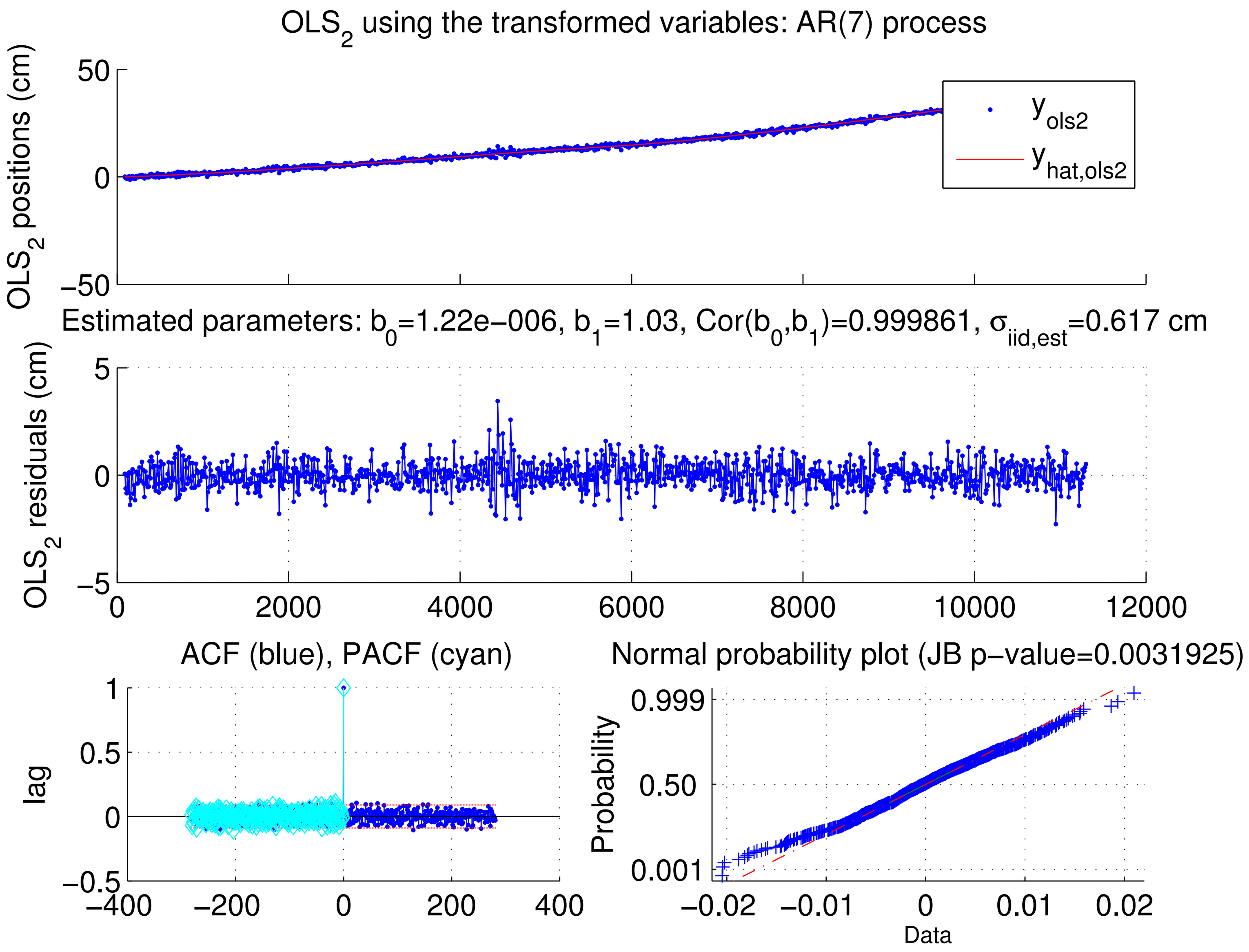
20 Correlated noise in GPS positions

- After applying the GLS transform, we obtain the "NG positions" as in 17
- Our estimate of noise of a few centimetres to be present in the real GPS positions is plausible
- But, the autocorrelation function (ACF) shows that the OLS residuals are correlated



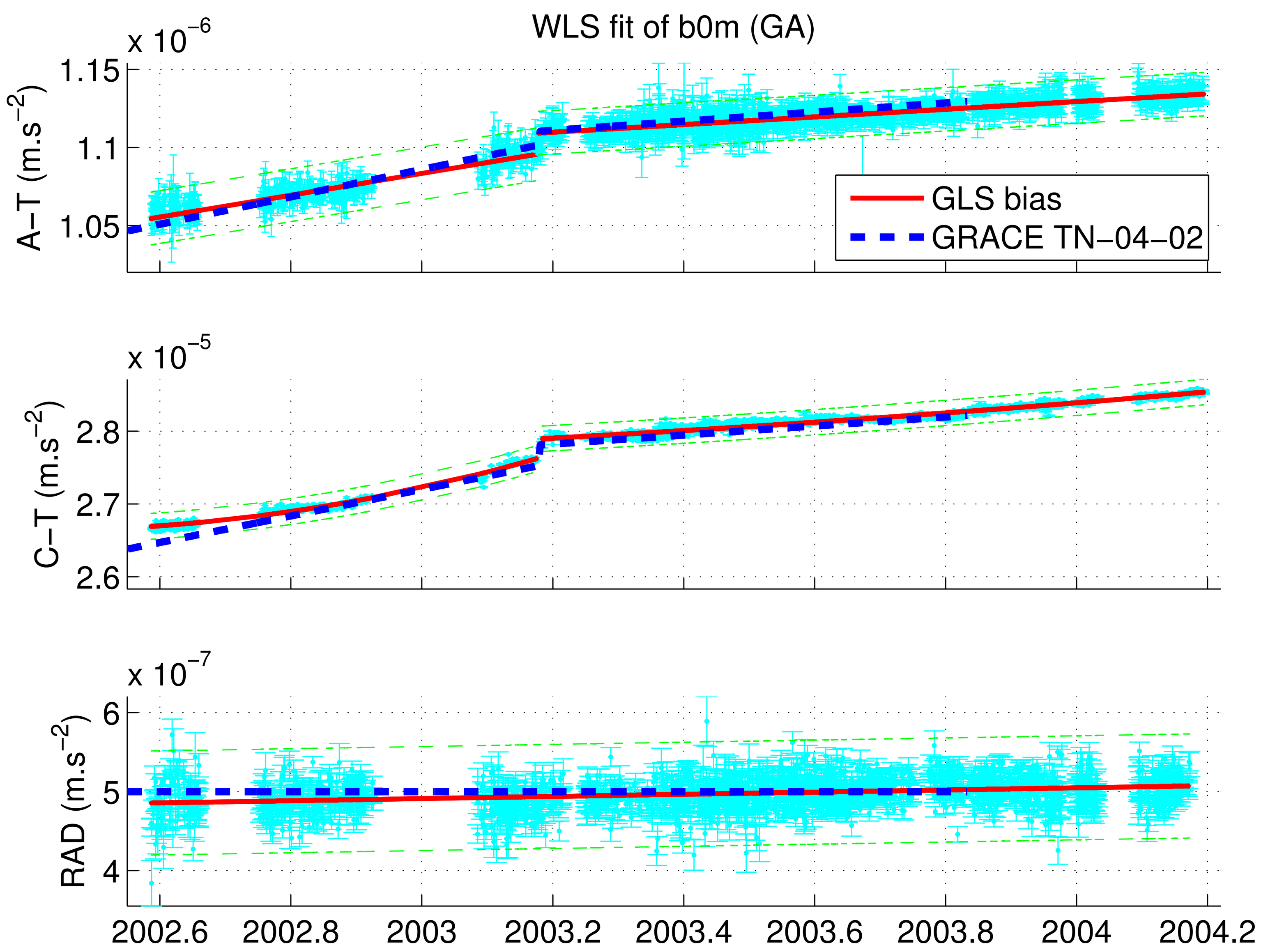
21 Removing the autocorrelation with an AR model

- Using the time series approach, we fitted the autocorrelation structure with an AR model
- The Cholesky decomposition of the autocovariance matrix of the fitted AR model provides the GLS transform matrix
- After the GLS transform, the residuals are again approximately uncorrelated ⇒ the calibration parameters and their uncertainties may now be obtained



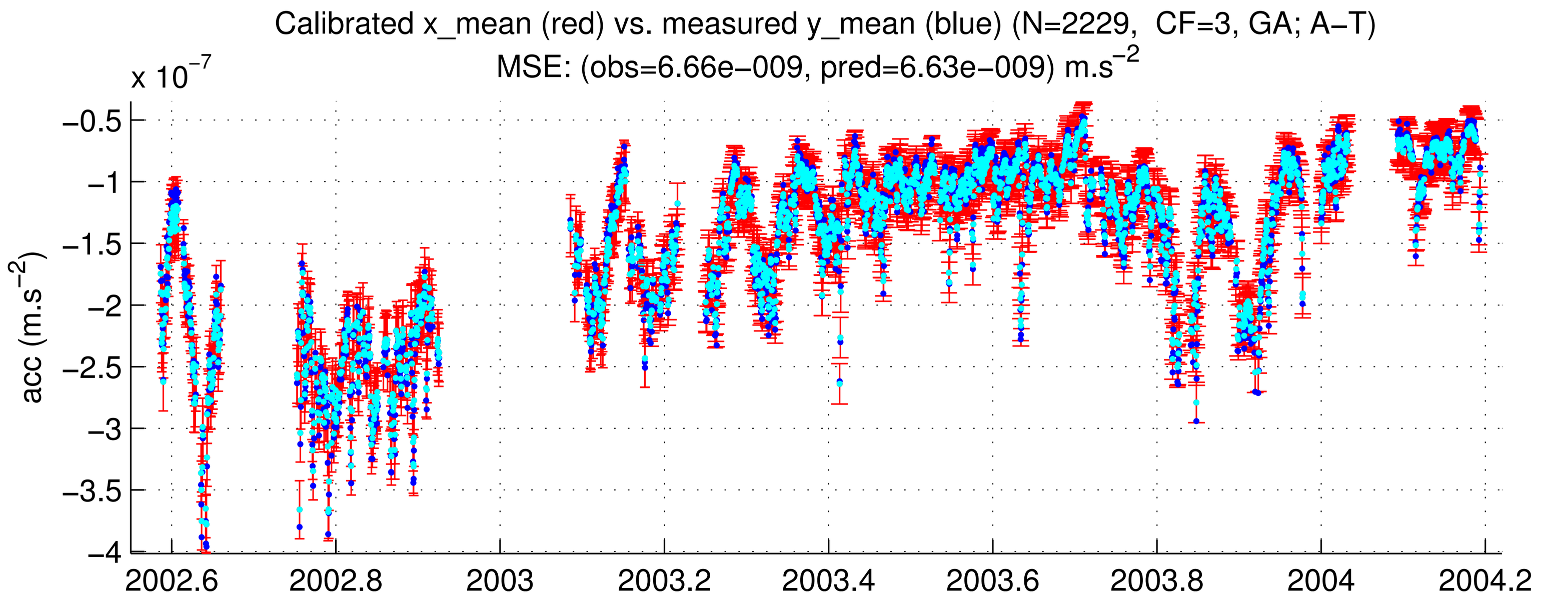
22 Calibration parameters over 1.5 years

- Due to a large bias B, the calibration parameters B and S are highly correlated
- We set the values of the scale factors according to GRACE TN-04-02 (Bettadpur, 2004) ⇒ bias offsets from our method then display a similar time evolution to those from the report



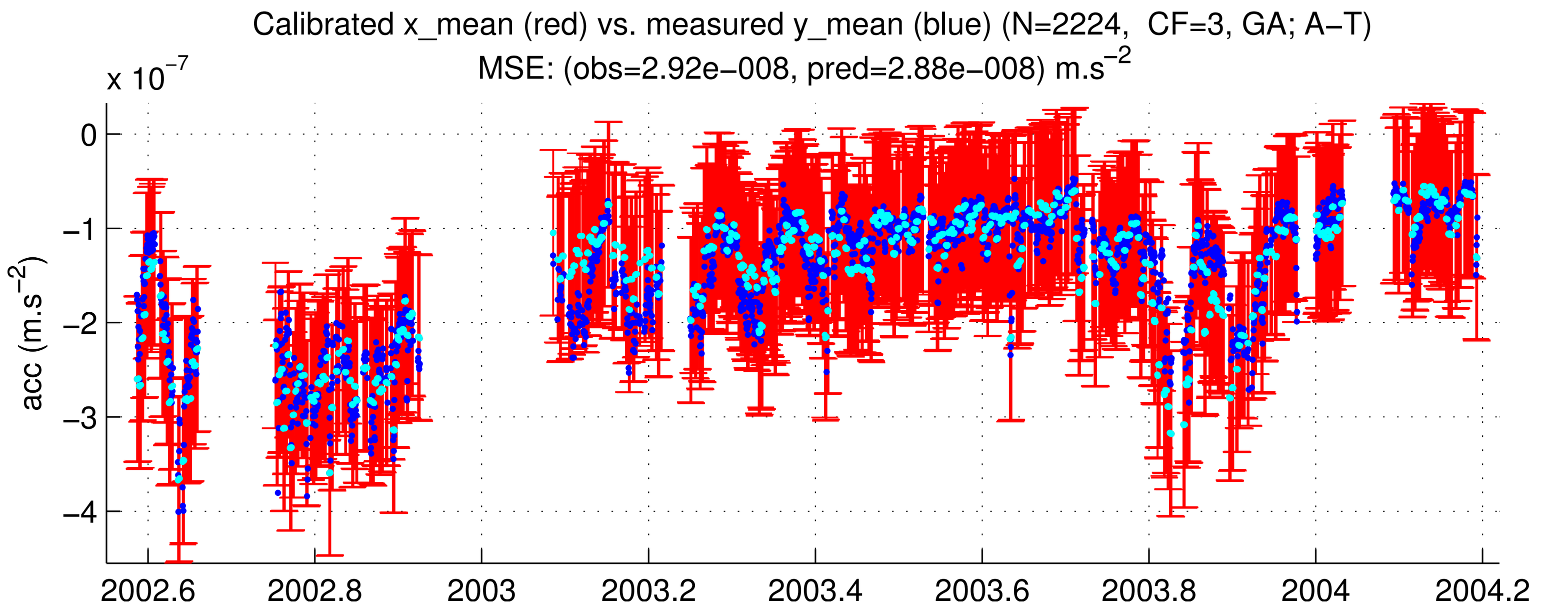
23 Calibrated ACC data over 1.5 years

- Shown are $a_{\text{NG}}^{\text{(CAL)}}$ for the A-T component



24 Calibration of simulated NG accelerations

- In the same way we also calibrated $a_{\text{NG}}^{\text{(SIM)}}$
- In the A-T component, an average error bar of the simulated NG accelerations is four times larger than that of ACC



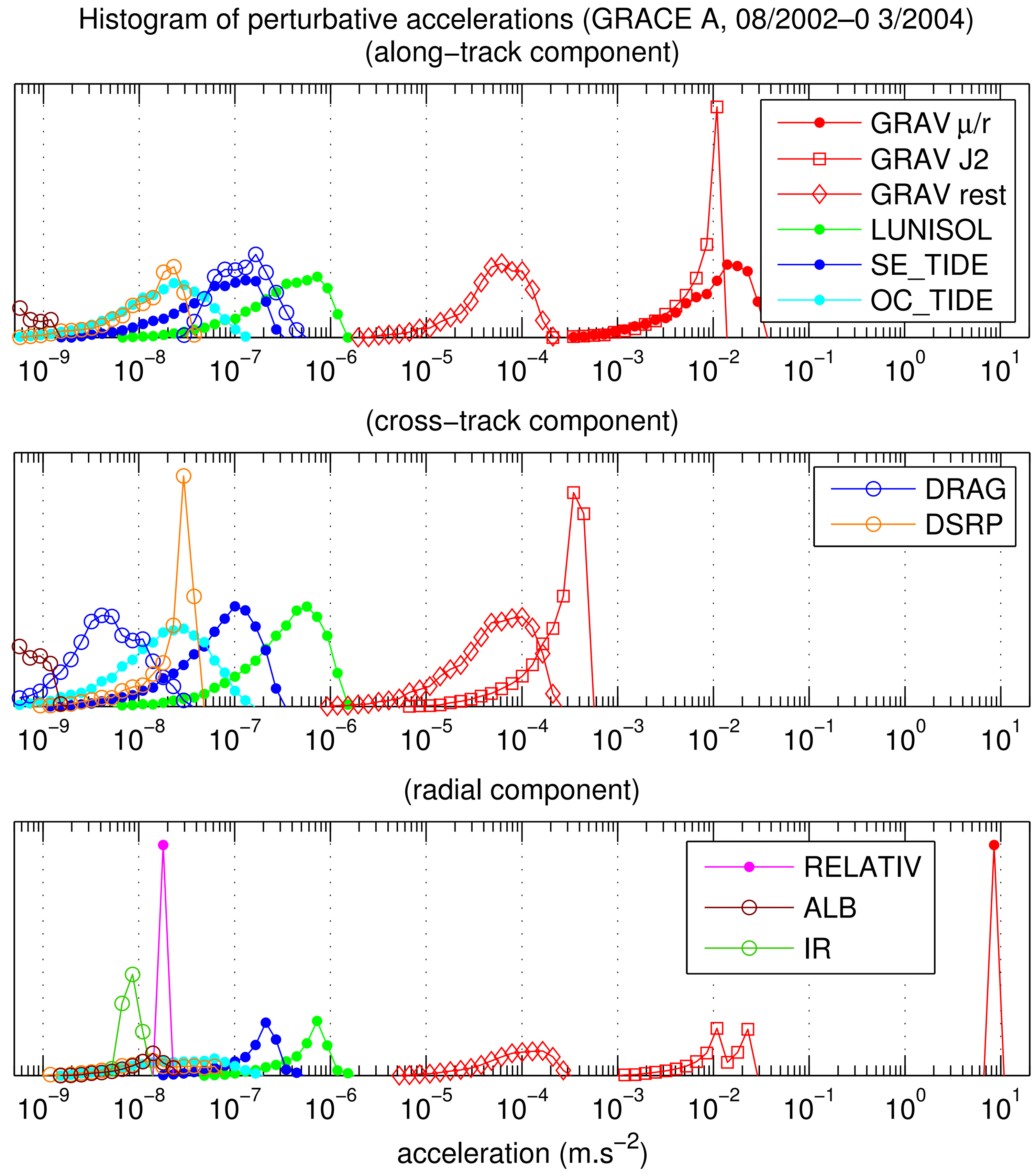
25 Performance of the method

- Results for GRACE A and B satellites are similar
- Usage of a running window of 2–5 revolutions makes no substantial difference
- The performance crucially depends on the quality of the modelled accelerations of static gravitational field
- Mean values over 1.5-year period are given
- Uncertainty in the A-T component is 3–8 times smaller for ACC than that of the simulated accelerations, but is similar in the C-T and RAD components

	Calibration	
	SIM to GPS	ACC to GPS
std error (m s^{-2}): A-T	$2.5–2.9\times10^{-8}$	$3.7–7.6\times10^{-9}$
std error (m s^{-2}): C-T	$2.8–3.6\times10^{-8}$	$2.8–3.9\times10^{-8}$
std error (m s^{-2}): RAD	$1.2–2.8\times10^{-8}$	$1.8–3.3\times10^{-8}$

26 Comparison with the magnitude of NG accelerations

- Over the period studied, the altitudes varied in 470–530 km
- Drag in A-T component: magnitudes 2×10^{-8}–3×10^{-7} m s^{-2}
  typical dayside value $\approx 2\times10^{-7}$ m s^{-2}
  typical nightside value $\approx 7\times10^{-8}$ m s^{-2}
- Ratio of the average A-T uncertainty to drag
  (as if all other error sources be zero)
  dayside ≈2–4 %
  nightside ≈5–11 %



27 Noise in GPS positions

- The estimated RMS values of noise in the GPS positions for the period of 1.5 years are in accordance with the quoted accuracy of kinematic orbits of 2–3 cm

	RMS of GPS noise	
	mean (cm)	std dev (cm)
A-T	2.3	0.9
C-T	3.8	0.6
RAD	4.9–5.2	1.8

Conclusions

Using the simulated positions, noise and accelerations, it was demonstrated that the method of generalized least squares (GLS) is capable of removing the autocorrelation and the high frequency amplification of random errors, thus providing statistically correct estimates of the calibration parameters and their uncertainties.

For the real-world data of GRACE A and B satellites covering 1.5 years, on using the state-of-the-art models of the geopotential and other forces, the GLS calibration method produced reasonable estimates of the noise level in the GPS positions. After removing the autocorrelation in the GPS positions noise through a fitted autoregressive model, the calibration parameters together with their accompanying uncertainties were obtained. Bias offsets and scale factors agree with those in an independent reference, thus providing additional evidence of the plausibility of the proposed calibration method.

Acknowledgements

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This is a slightly extended version of the poster