Simulation of free fall and resonances in the forthcoming GOCE mission

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Abstract

GOCE, ESA's first Earth gravity mission, was launched on 17 March 2009 into a sun-synchronous orbit. Using the full-scale numerical propagator, we investigated the satellite's free fall from the initial injection altitude of 280 km down to the first measurement phase altitude (at 264 km). During this decay phase the satellite will pass below the 16:1 resonance (268.4 km). The effect of this resonance, together with the uncertainty in the solar activity prediction, has a distinct impact on the evolution of the orbital elements. Then, to maintain a near-constant and extremely low altitude for the measurement operational phases, the satellite will use an ion thruster to compensate for the atmospheric drag. In order to obtain the groundtrack grid dense enough for a proper sampling of the gravitational field, ESA set constraints for a minimum groundtrack repeat period. We studied suitable repeat cycles (resonant orbits) in the vicinity of 16:1 resonance; we found that they differ greatly in stability towards small perturbations of the satellite's mean altitude and in temporal evolution of the groundtrack coverage. The results obtained from the usual analytical treatment of orbital resonances were refined by more realistic numerical simulations. Finally, we formulated suggestions that might be useful in GOCE orbit planning.

Introduction

ESA's GOCE satellite

- To date the most advanced gravity space mission
- Successfully launched on 17 March 2009
- The main scientific instrument: gradiometer
- Electric ion propulsion system to counteract drag
- Goal of the mission: to improve the global and regional models of the Earth gravitational field

Orbit of GOCE

- Dusk-dawn nearly sun-synchronous orbit
- Very low circular orbit

Topics of our study

- Free fall in the early orbit phase
- Suitable groundtrack (GT) repeat orbits for the measurement operational phases (MOP)

Free fall of GOCE

- Controlled decay from injection altitude 280 km down to MOP1 altitude 263 km
- Correction of potential launch injection errors
- Check out of gradiometer

Aim of our simulation

- Modelling all the perturbing accelerations
- Reliable prediction of time needed for the satellite to descend to MOP1

Manifest features of graphs

- Atmospheric drag (Fig. 1) ⇒ steady decrease in semimajor axis (SMA) or in mean altitude
- When the satellite is tilted ('15° tilt'), the drag is increased w.r.t. the 'nominal' satellite attitude
- 'Max'/'min' predicted level of solar activity (11-year sunspot cycle) ⇒ increased/decreased atmospheric drag
- Passage through strong 16:1 resonance ⇒ quasi-secular changes especially in inclination

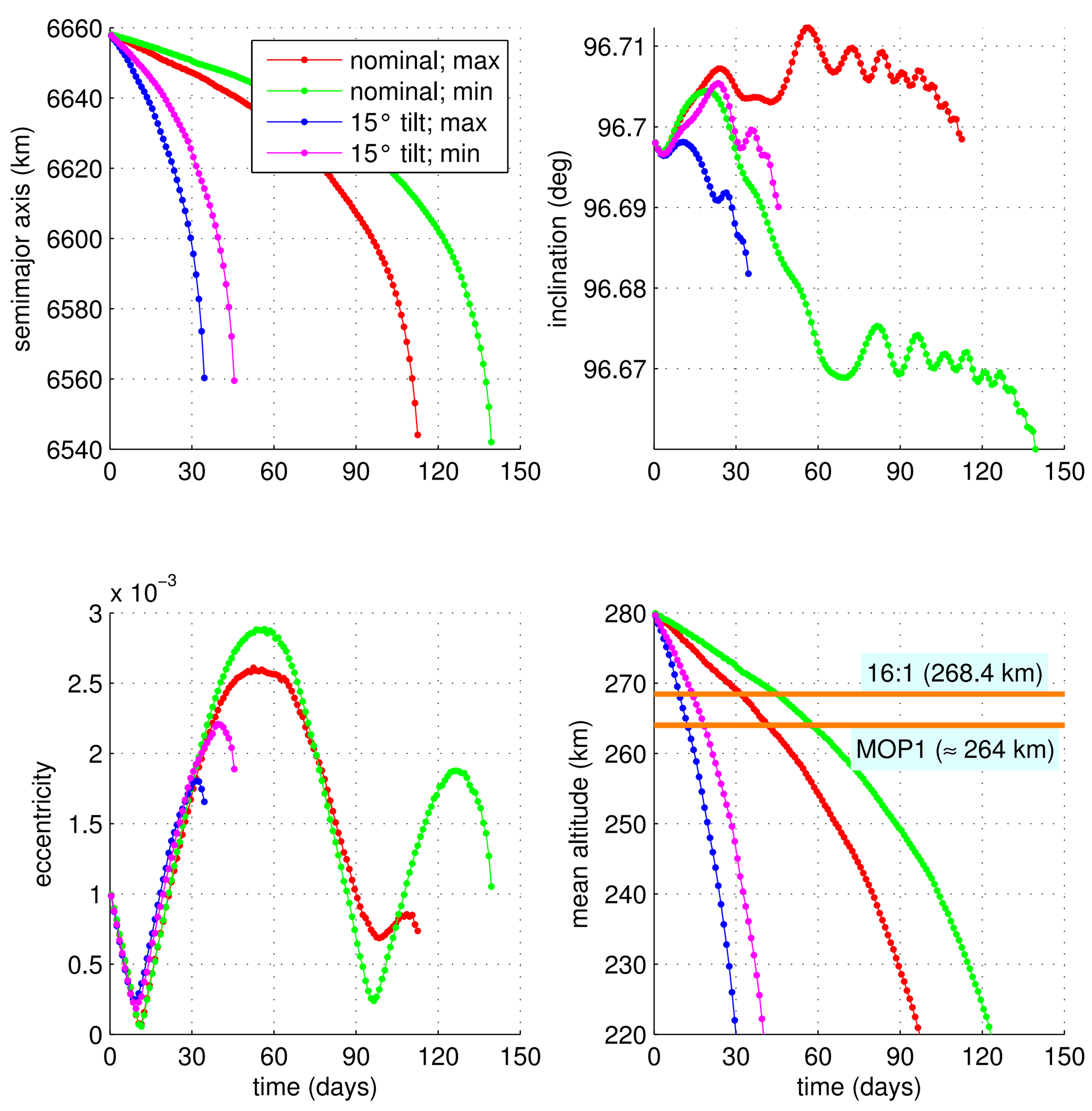

Figure 1. Mean orbital elements calculated by NUMINTSAT.

Deep solar minimum

- This is the quietest sun we've seen in almost a century. (Science@NASA, April 1, 2009)
- A delayed start of the new cycle of solar activity ⇒ MOP1 is planned below the 16:1 resonance

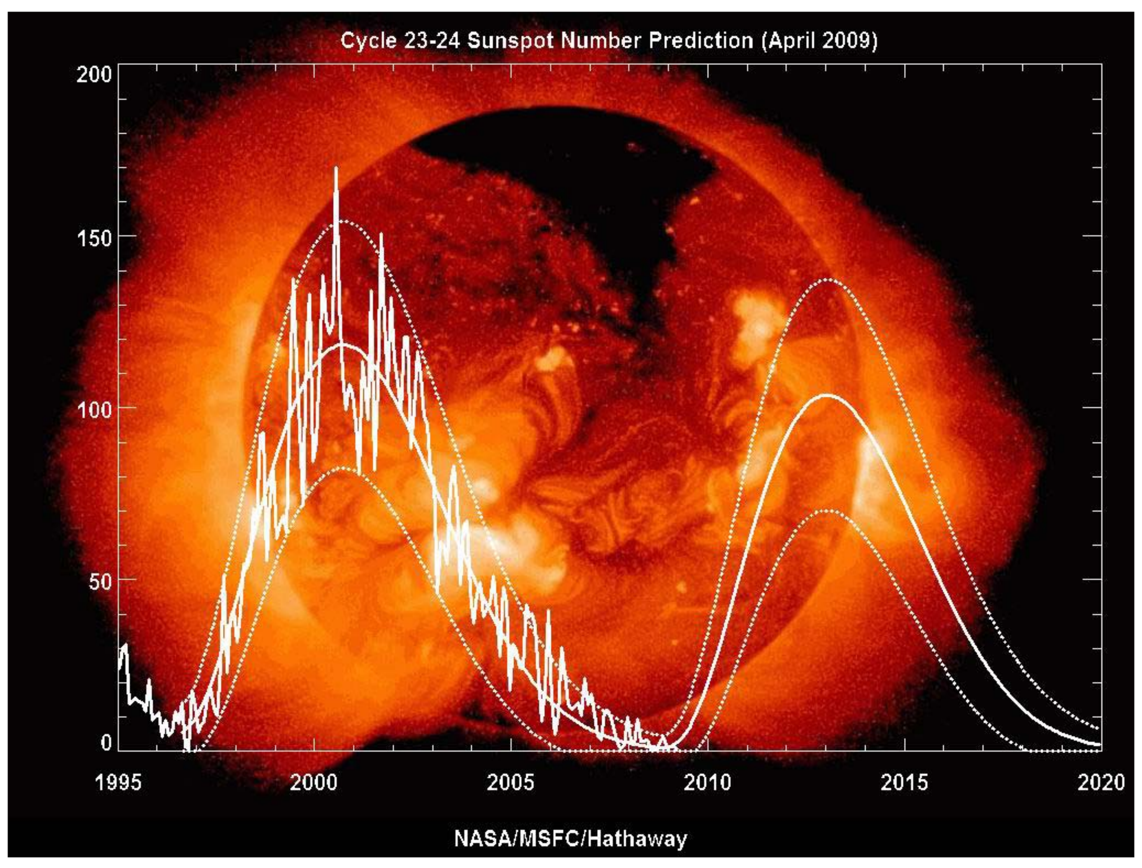

Figure 2. Solar activity cycle.

Orbital propagator NUMINTSAT

- Our 'home-made' propagator for LEO satellites
- In Fig. 3, absolute values of simulated accelerations acting on GOCE

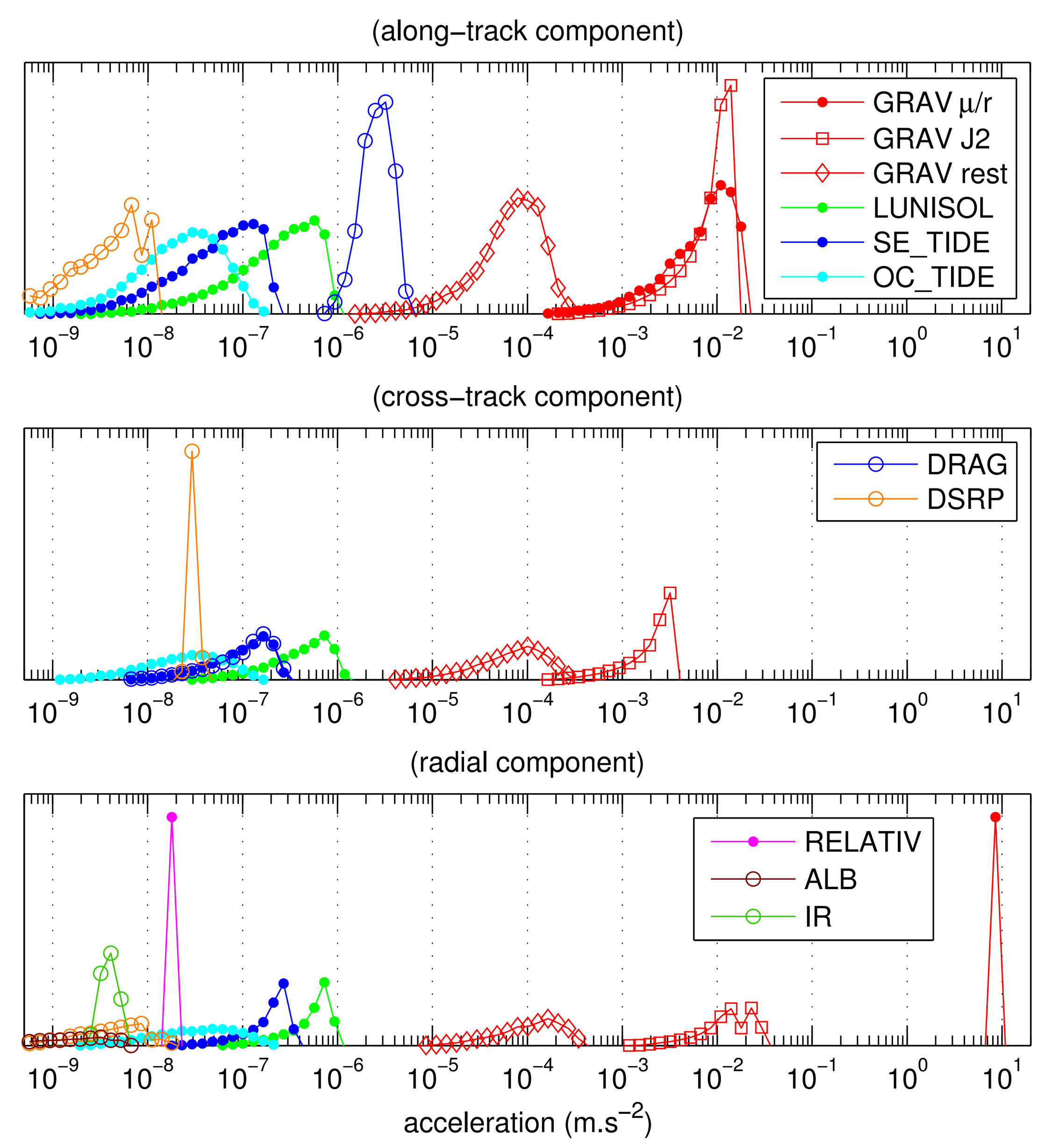

Figure 3. Histograms of gravitational and nongravitational accelerations acting on GOCE.

Resonances and GT coverage

Orbital resonance R:D – the satellite performs R nodal revolutions, while the Earth rotates D times w.r.t. satellite's precessing orbital plane.
Groundtrack repeat orbit – a GT that repeats after an integer number R of orbital revolutions and an integer number D of nodal days.

Use of resonant (or repeat) orbits

- Evaluation of lumped geopotential coefficients
- Mission planning for Earth observing satellites (e.g. satellite altimetry)

Analytical treatment of orbital resonances

- Effects of only J_2 perturbation (oblateness)
- Expressions for nodal revolution, $2\pi/(\dot\omega+\dot M)$, nodal day, $2\pi/(\omega_e-\dot\Omega)$, ω_e …Earth angular rate, and secular changes due to J_2:
$$\dot\Omega = -\tfrac{3}{2}nJ_2\left(\tfrac{R_e}{a}\right)^2\cos i\,(1-e^2)^{-2}\,,$$
$$\dot\omega = -\tfrac{3}{4}nJ_2\left(\tfrac{R_e}{a}\right)^2(1-5\cos^2 i)\,(1-e^2)^{-2}\,,$$
$$\dot M = n - \tfrac{3}{4}nJ_2\left(\tfrac{R_e}{a}\right)^2(1-3\cos^2 i)\ (1-e^2)^{-3/2}\,,$$
combine to give the following
- Resonance condition for mean motion
$$n = \omega_e\tfrac{R}{D}\left\{1-\tfrac{3}{2}J_2\left(\tfrac{R_e}{a}\right)^2\left(4\cos^2 i-\tfrac{R}{D}\cos i-1\right)\right\}$$
for given $e \simeq 0$ and $i \Rightarrow R, D, a$

Example: CHAMP, GRACE

- Gravity field modelling requires dense enough sampling of Earth surface
- Degradation in accuracy of GRACE monthly gravity solutions near 61:4 (9/04)
- CHAMP passed through 46:3 (10/00), 31:2 (5/02, 10/02, 6/03), 47:3 (11/05, 1/07)

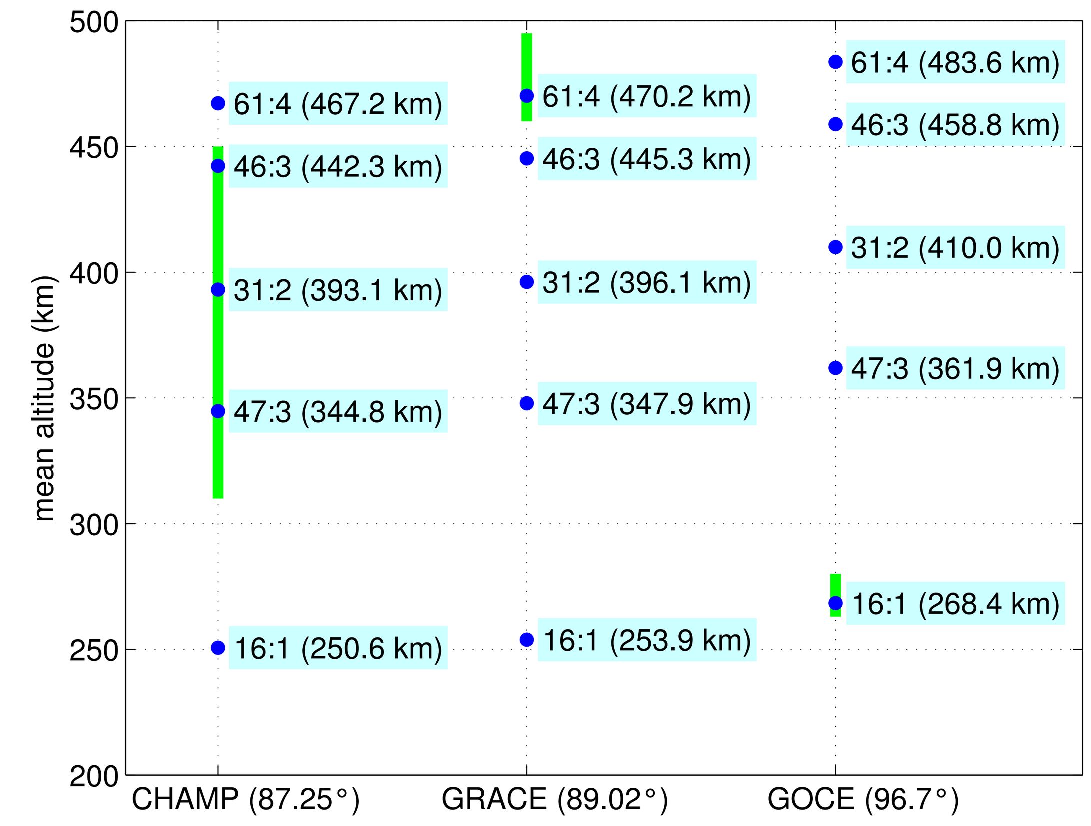

Figure 4. Main orbital resonances of current geodetic missions (relevant height range in green).

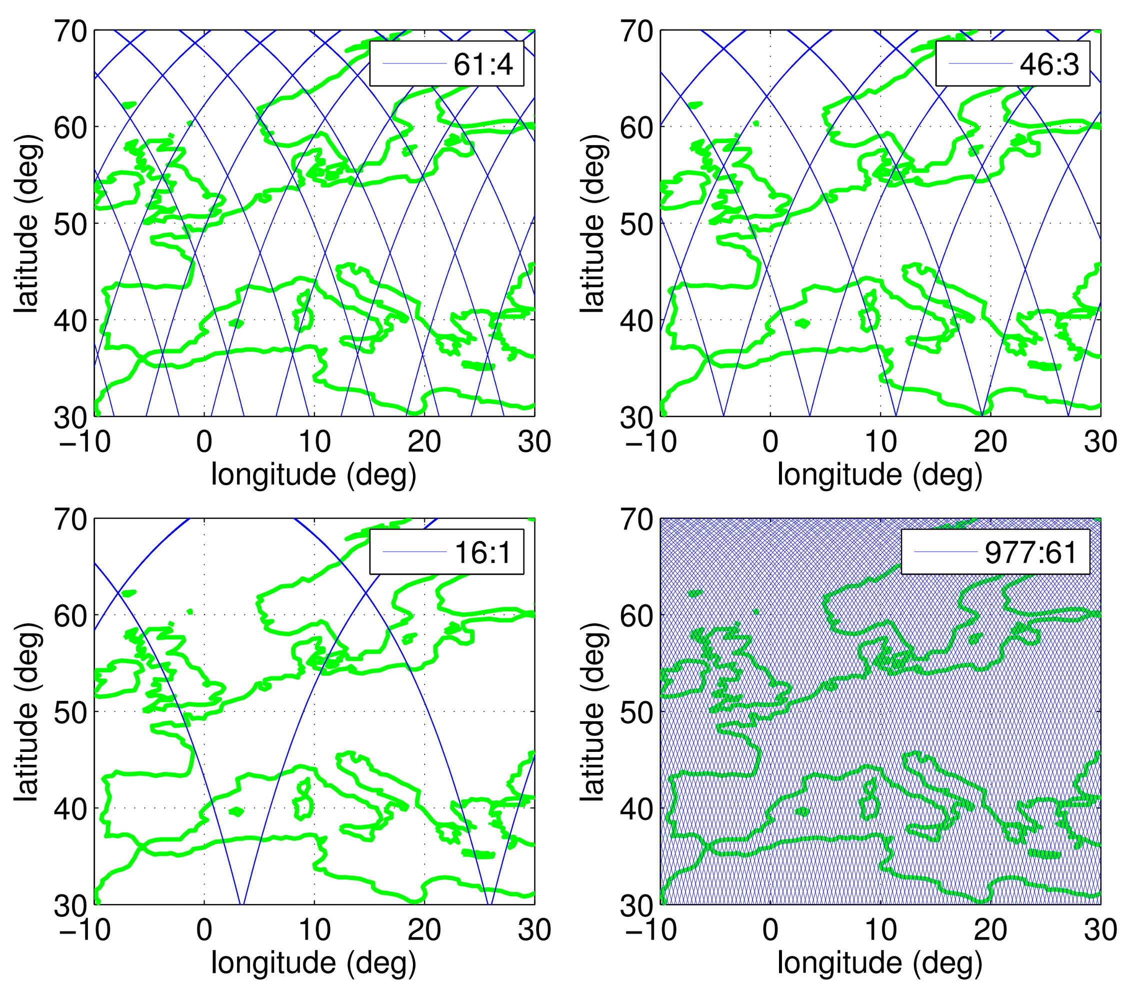

Figure 5. GT coverage for different resonant conditions.

GOCE

- A global uniformly distributed dense coverage $\Rightarrow$ GT repeat period $\geq$ two months
- Sun-synchronicity $\Rightarrow$ inclination 96.7°
- Resonance diagram with two 61-day repeat orbits, candidates for MOPs

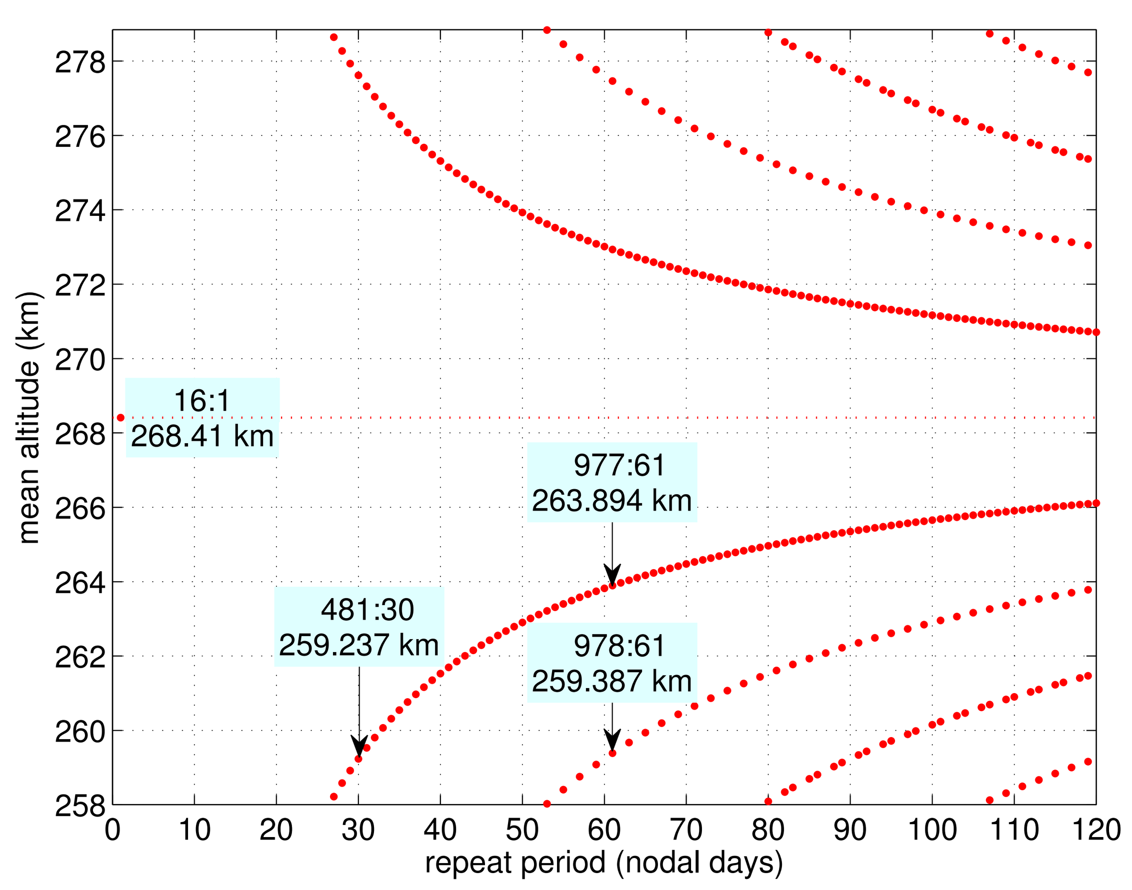

Figure 6. Orbital resonances predicted for GOCE (i=96.7°).

Equatorial node separation, $\Delta\lambda$

- After the completed repeat period, GT grid should be homogeneous with an equatorial node separation $\Delta\lambda = 360°/R$
- After 61 days, $\Delta\lambda^{(977)}$=0.3685° (41.02 km) and $\Delta\lambda^{(978)}$=0.3681° (40.98 km) $\Rightarrow$ similar GT grid

Evolution of groundtrack (GT) coverage

- 977:61 – consecutively filling up the equator during the whole repeat period
- 978:61 – created in two 30-day phases, the half-filled grid being close to 481:30 orbit

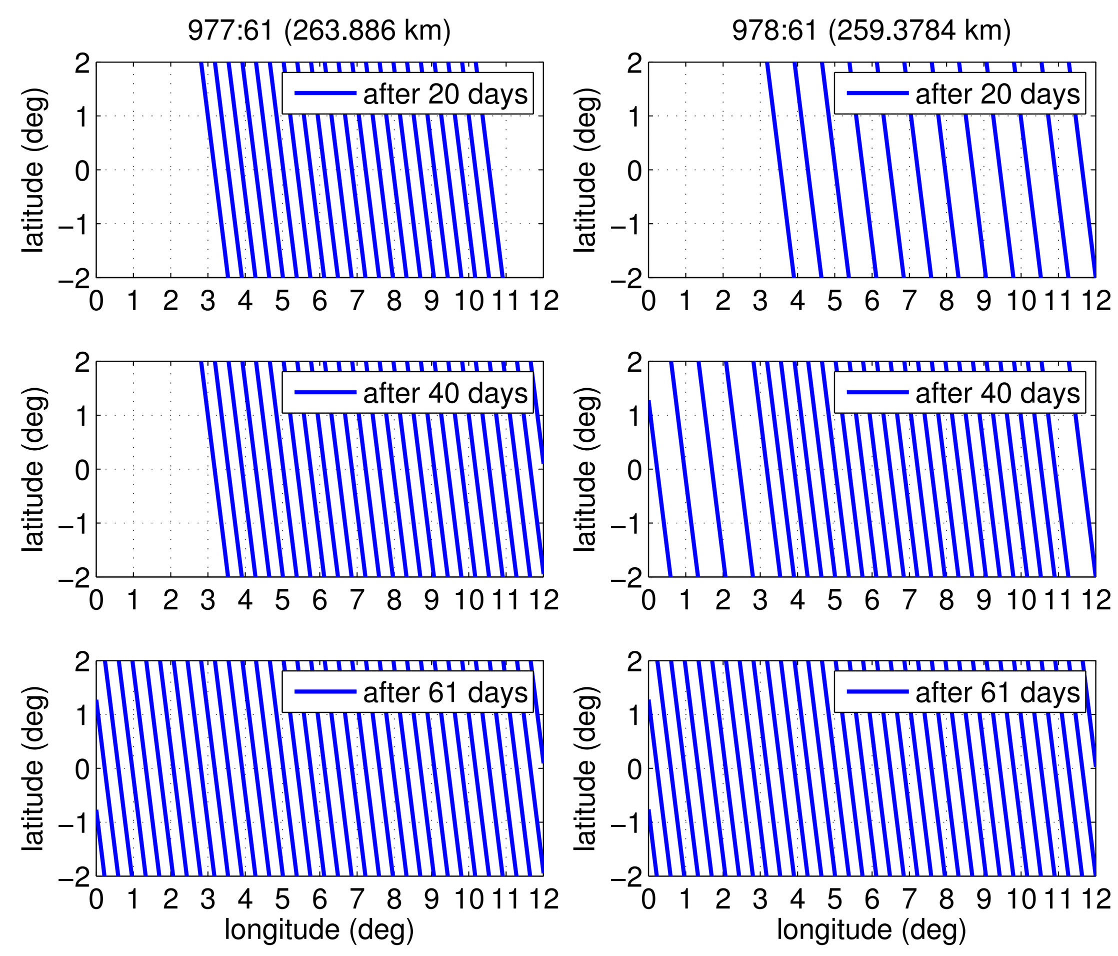

Figure 7. Evolution of GT grids for 977:61 and 978:61 orbits.

Need of SMA adjustment to obtain GT repeat

At first, histograms of node separation for 978:61 orbit produced a double peaked graph – but such a histogram should produce a *single* peak, maybe spread around the central value 360°/R.

Histograms based on analytical theory: 978:61

- Recall: For GOCE we need no equatorial gaps larger than 42 km (0.377°)
- After 61-day repeat period, we draw the histograms of lengths of the gaps
- Blue bar – the node separation $\Delta\lambda\simeq$0.368° corresponding to an exact 978:61 repeat
- Altitude reduced by 50 m $\Rightarrow$ two green bars, etc.
- 150-m height reduction $\Rightarrow$ histogram in cyan with 0.748° corresponding to 481:30 orbit
- Only a 50-cm disturbance in height shows the instability of 978:61 orbit

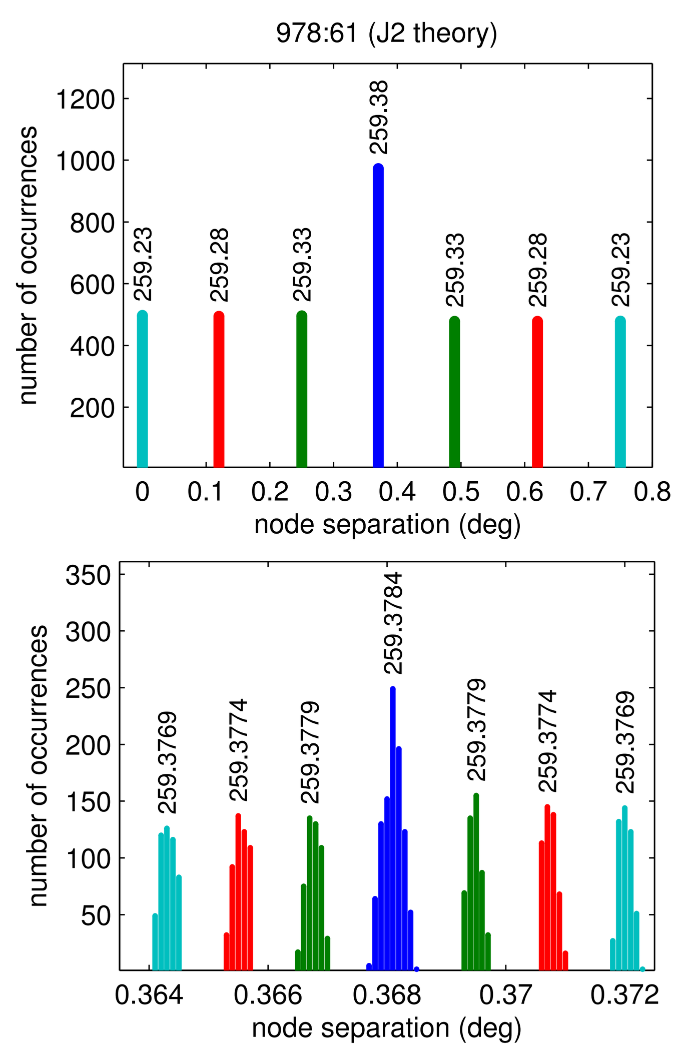

Figure 8. Histograms of node separation for orbits near the 978:61 resonance as function of the mean altitude.

Histograms based on analytical theory: 977:61

- Repeat character is preserved, even if altitude is varied, e.g., by ±100 m or ±200 m

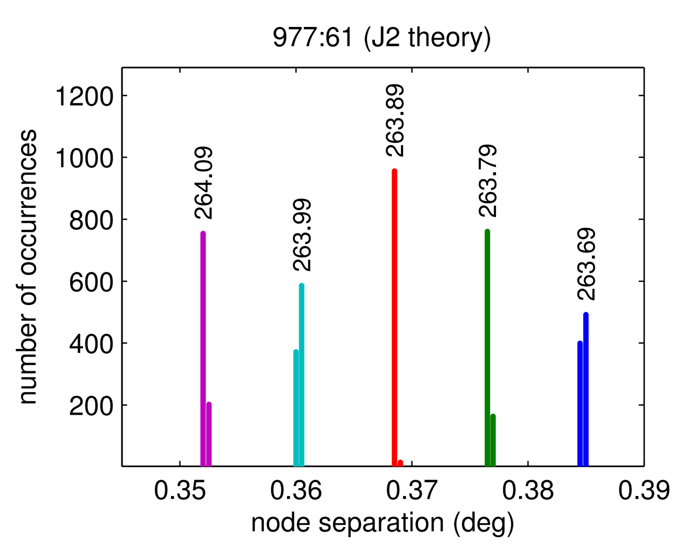

Figure 9. Histogram of $\Delta\lambda$ for orbits near 977:61.

Histograms from numerical integrator: 977:61

- We took into account other perturbations (Fig. 3)
- Narrow bars from J_2 theory become wider, but still the bars are single-peaked around $\Delta\lambda^{(977)}$
- Altitude 263.9 km from the J_2 theory is still valid

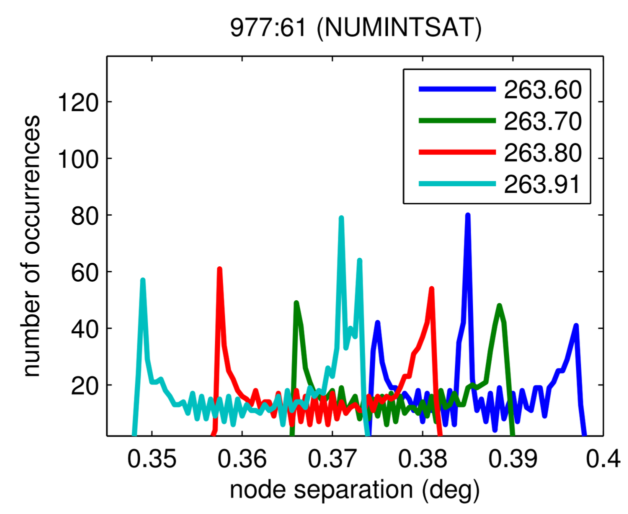

Figure 10. Histogram of $\Delta\lambda$ for orbits near 977:61 resonance using the full numerical integrator.

Conclusions/suggestions for GOCE

Free fall phase

- 16:1 resonance may induce inclination changes ±0.03° $\Rightarrow$ re-adjust SMA to fulfil the orbit requirements *after* each pass through resonance
- After adoption of higher value for the drag coefficient, C_D, our prediction of descent times gives comparable results to those of ESA

Study of two suitable 61-day repeat orbits

- 977:61 orbit vs. 978:61 orbit, lower by 4.5 km
- GT grids almost the same with $\Delta\lambda\simeq$41 km
- GT grid of 977:61 laid down consecutively, that of 978:61 in two 30-day subgrids
- 978:61 orbit loses repeatability already by 50-cm variation in height, while 977:61 orbit retains it $\Rightarrow$ **first branch below 16:1 resonance preserves better the repeatability conditions**
- The same applies to first branch *above* 16:1
- Results were tested using both analytical and full numerical orbital propagators

Current orbital data

Planned mission phases

Comm.	@273 km,	Apr 2009	–	Aug 2009	(5 months)
MOP1	@263 km,	Sep 2009	–	Mar 2010	(7 months)
HOP1	@273 km,	Apr 2010	–	Aug 2010	(5 months)
MOP2	@263 km,	Sep 2010	–	Mar 2011	(7 months)

Planned altitudes lie on the first branch below and above 16:1 resonance, in accord with our results

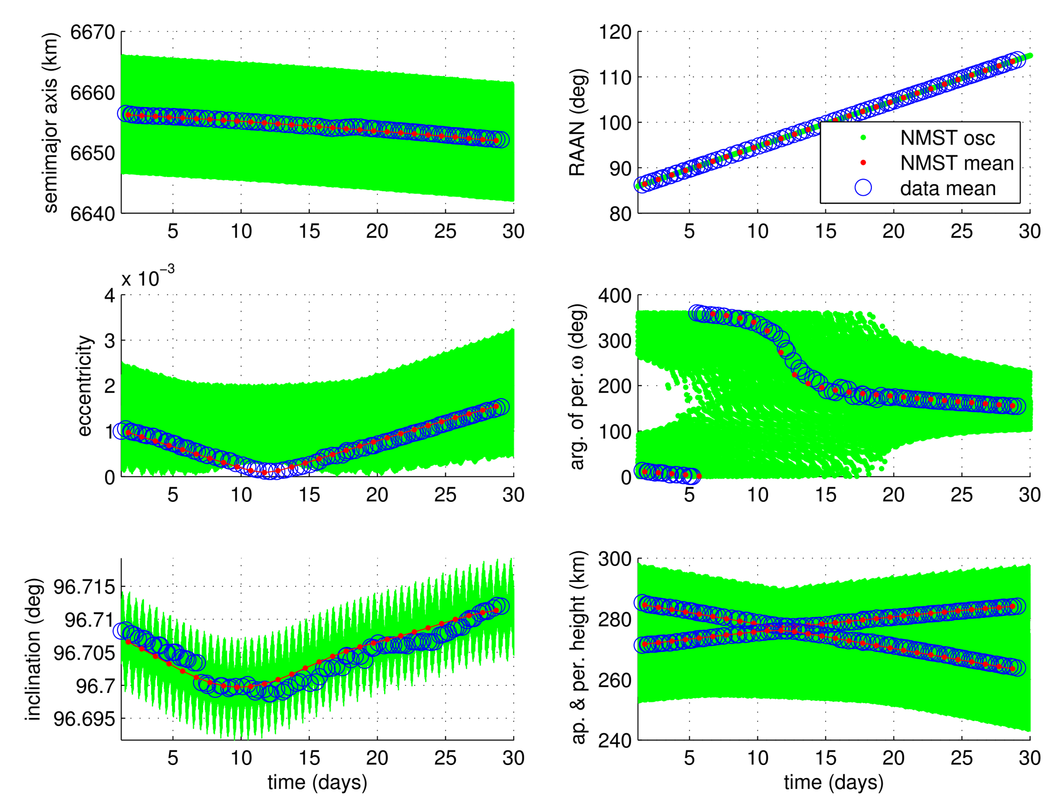

Figure 11. Orbital elements calculated by NUMINTSAT.

Reference

Bezděk et al., 2009. Simulation of free fall and resonances in the GOCE mission. J. Geodyn., in press, doi: 10.1016/j.jog.2009.01.007