SOME ASPECTS OF THE ORBIT SELECTION FOR THE MEASUREMENT PHASES OF GOCE

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ABSTRACT

For the satellite GOCE, ESA's first Earth gravity mission equipped with a gradiometer and launched in March 2009, the choice of the orbit for the measurement phases is a key ingredient for the successful accomplishment of the mission's aims. To maintain a near-constant and very low altitude around 245–280 km, GOCE will use an ion thruster to compensate for the atmospheric drag. In order to obtain the groundtrack grid dense enough for a proper sampling of the gravitational field, ESA set constraints for a minimum groundtrack repeat period. We compared suitable repeat orbits from several aspects: temporal evolution of the groundtrack coverage; homogeneity of the groundtrack grid; stability with respect to variations in the mean altitude; proximity in altitude of orbits with shorter repeat periods (cycles and subcycles of the repeat orbits). While the analytical computations are capable of showing a broad picture of the possible orbital configurations, the precise simulations based on numerical orbit integration accounting for various perturbations allow us to include the most realistic orbital conditions, which have to be taken into account for the final decision about the orbit choice and fine orbit tuning (small changes in height of satellite orbit). We formulated suggestions that might be useful in the planning of the forthcoming measurement operational phases of GOCE gradiometer.

1. INTRODUCTION

The broader context and the details of the techniques used to obtain the results presented in this paper, together with a more complete list of references, can be found especially in [1]. We also applied the presented methodology of the fine orbit tuning for the accuracy gain in the gravity field modelling to satellite orbiters around other planetary bodies of the Solar System [2]. The analysis of orbit resonances for recent satellite geodetic missions can be found e.g. in [3].

Guided fine orbit tuning for gravity field missions, with the aim to improve the accuracy of the gravity field parameters derived from the measurements of those missions, is now feasible as a tool to utilize in an optimum way the precision and resolution of the instrumentation available on board: In this case we discuss the gradiometer on GOCE.

1.1. Orbital resonances

Orbital resonance R:D takes place, when the satellite performs exactly R revolutions with respect to its ascending node, while the Earth rotates exactly D times with respect to the precessing orbital plane (R and D being coprime integers). The time between two consecutive passages of the fixed Earth meridian over the satellite ascending node is sometimes called a *nodal day*. In other words, in an orbital resonance, the groundtracks are exactly the same after R nodal revolutions and D nodal days. If we take into account only the gravitational forces (and neglect other perturbations, mainly atmospheric drag), the groundtrack grid as shown in Fig. 1 will not change with time, after the repeat period is over, the satellite is flying over the same subsatellite points on the Earth. In the sequel, we will use the terms *repeat orbit* and *resonant orbit* interchangeably.

Figure 1. Satellite in 15:1 orbital resonance.

In Fig. 2 it is illustrated that the *lower* the order of the resonance R, the *sparser* the groundtrack grid is, after the particular repeat period has been completed. Based on the Nyquist sampling theorem, one can intuitively see that to recover the longitudinal sinusoidal part of the spherical harmonic series of the geopotential, one would need at least two data points per period for a given spherical frequency. This leads to the approximate rule that if the satellite is in the R:D repeat orbit, the

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maximum degree/order n_{max} of the Stokes parameters that might be fully recovered is given by (e.g. [4])

$$n_{max} \leq R/2. \tag{1}$$

Another question is resolvability with a lower quality which can be achieved till R, but not above it. This topic has been discussed in [4] and re-opened e.g. in [7].

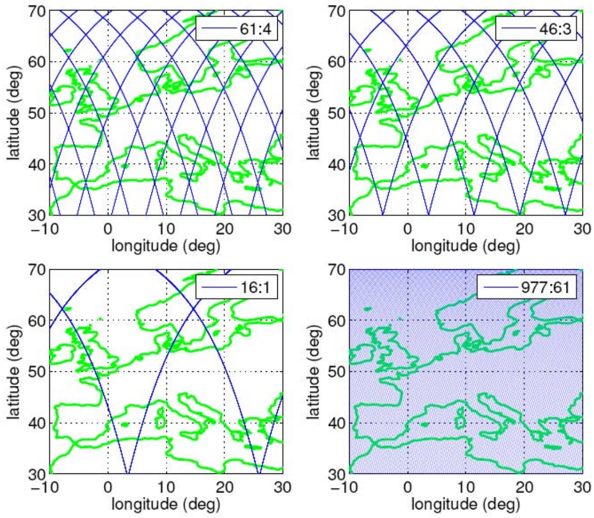

Figure 2. Example groundtrack grids of several low order resonances.

1.2. Recent satellite gravity missions

The international space geodesy community lives now a “golden age” with several satellite missions fully dedicated to the study of the Earth gravity field. This started with the launch of the German CHAMP satellite in 2000 (http://isdc.gfz-potsdam.de/), followed by two GRACE satellites operated by DLR/NASA in orbit since 2002 (http://www.csr.utexas.edu/grace/), now we are waiting for the first official results from GOCE , launched by ESA in 2009 (http://www.esa.int/SPECIALS/GOCE/).

Fig. 3 shows passages of these satellites through the strong orbital resonances. In the present context, the most important is the passage of GRACE through the 61:4 resonance, which attracted attention to the influence of the geometry of orbit resonances on the uncertainty of the obtained geopotential harmonic parameters (in that case for the monthly gravity solutions).

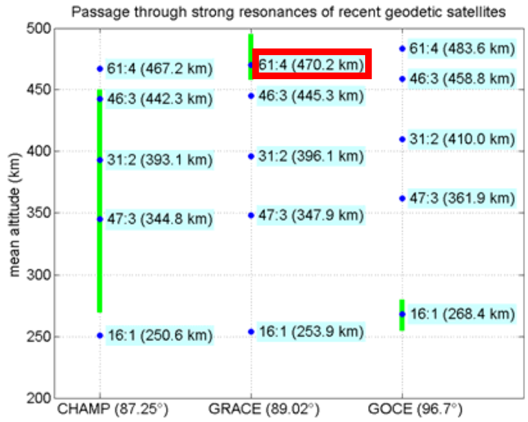

Figure 3. Passage through strong resonances of recent geodetic satellites. The altitude range, through which the particular satellite has already passed, is in green.

In Fig. 4 it is shown on the example of the NASA/GSFC monthly GRACE solutions (http://grace.sgt-inc.com/) that in August–September 2004 the precision of the obtained geopotential parameters is visibly worse (note the logarithmic scale of the y-axis) than before and after that. Why this happened was not clear at the beginning, but the explanation has quickly been discovered: the decrease in the accuracy is due to the passage of GRACE through the 61:4 resonance (see e.g. [4]). Indeed, in this case the grid of groundtracks is rather sparse (upper left graph in Fig. 2) and does not allow the gravity field solutions to be as precise for orders/degrees around 60 (or even 30 according to the R/2 rule).

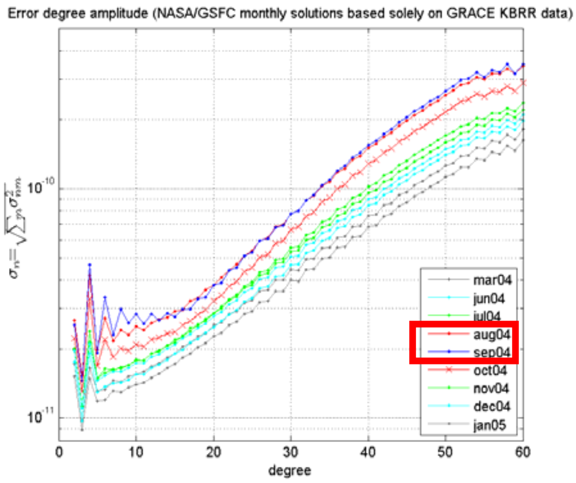

Figure 4. Error degree amplitude of monthly solutions from GRACE around the passage through the 61:4 resonance in August-September 2004.

2. GOCE AND ORBITAL RESONANCES

"The primary GOCE scientific mission objective is to provide a global model of the Earth's gravity field and the geoid with high spatial resolution and accuracy. More specifically, after ground processing, the goals are to determine the Earth's gravity field and its anomalies with an accuracy better than 1 mGal, and the global geoid with an accuracy better than 1–2 cm. Both these goals should be achieved at a spatial resolution of 100 km (half-wavelength) or better, corresponding to a spherical harmonic expansion up to degree and order 200. " [5] This is the reason for the ESA's constraint of a minimum two-month repeat period. Due to the attenuation of the gravitational field with altitude, the GOCE height should be as low as possible depending on the performance of the onboard ion thruster, which acts against the atmospheric drag (mostly along-track) and maintains the constant altitude during the MOPs.

In Fig. 3 it is shown for GOCE that from its initial altitude of 280 km on its descent to the first measurement phase, the satellite passed through the 16:1 resonance, which noticeably affected the orbital elements of the satellite, mainly its inclination [1]. Due to the sparsity of the groundtracks of this repeat configuration (lower left graph in Fig. 2) it has to be avoided for the measurement phases.

In [1] we studied two candidate 61-day repeat orbits, which provide dense enough sampling (lower right graph in Fig. 2) for the MOPs, a higher orbit 977:61 with no subcycles (orange box in Fig. 5) and a lower orbit 978:61 with a 30-day subcycle orbit 481:30 (green boxes in Fig. 5).

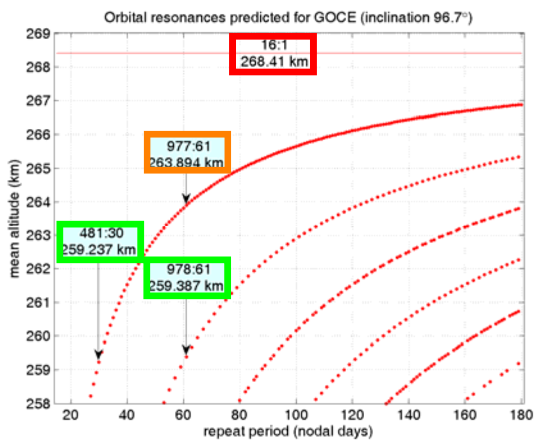

Figure 5. Resonance diagram for the GOCE satellite.

It is important to note that some orbits have not any subcycles while others have them. In Fig. 5, the orbits without any subcycle are on the highest bough of orbits just below the 16:1 orbit. Evolving of the ground tracks and their density differ dramatically for the orbit without and with subcycles, which we show in Fig. 6.

2.1. Temporal evolution of an orbit – with/without a subcycle

The main difference between the repeat orbits with and without a subcycle is the way, how the Earth is covered by the grid of groundtracks. In the case of the repeat orbit with no subcycle (Fig. 6, left panels), the orbital plane is slowly drifting with respect to the Earth surface and the two large gaps between the satellite ascending and descending nodes are gradually filled up. The grid is completed after the repeat period is over. This is different for a repeat orbit with a subcycle (Fig. 6, right panels). Here, after the *subcycle* period has elapsed, the surface is covered with a "half-dense" grid, which is made denser due to measurements in the following subcycle period to provide the complete grid at the end of the full repeat cycle. Thus the advantage of the repeat orbit with subcycles is that one obtains a less dense, but homogeneous coverage of the Earth surface quickly, already in the middle of the full repeat cycle. This might be useful in cases of an unforeseen failure of the onboard thrusters etc. The advantage of the repeat orbit with no subcycles is that they are more stable with respect to small changes in the mean altitude (more about it in [1] and in the next section).

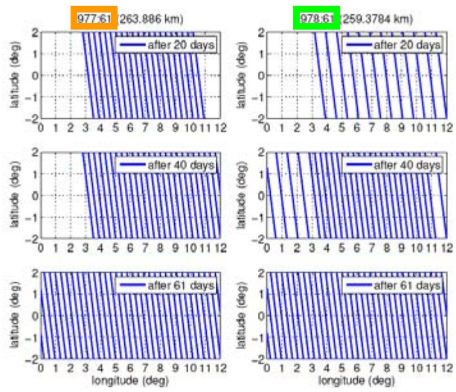

Figure 6. Temporal evolution of a repeat orbit without a subcycle (on the left), and with a subcycle (on the right).

2.2. Small variations in altitude of repeat orbits

The evolution of the groundtrack grid of the exact 61-day repeat orbit is shown by the left column of panels in Fig. 7. After the 61-day repeat period is over, we may look at the groundtrack grid (formed only by the ascending nodes for simplicity) at the equator, as is shown in the lower panels of Fig. 7. The histogram of these *equatorial node separations* (ENS) for the exact 61-day repeat orbit is represented by the single blue peak at the centre of Fig. 8.

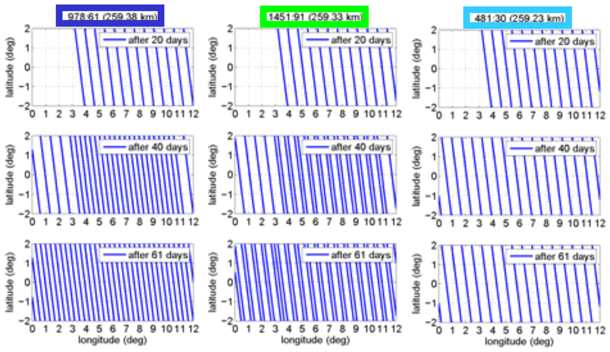

Figure 7. Difference in Earth coverage of orbits after 61 days, whose mean altitudes are rather close.

If the mean height 259.38 km of the 61-day orbit is decreased by 50 metres, the groundtrack grid changes and is not regular any more after 61 days (middle column of panels in Fig. 7). The corresponding histogram in Fig. 8 displays two peaks (green boxes) and the orbit can identified as the 91-day repeat one (on the third branch of the resonance diagram in Fig. 5 with approximately the same altitude as the 30-day and 61-day orbits). If the mean height is lowered further in total by 150 m, we obtain the groundtrack grid of the 30-day repeat orbit (Fig. 7, right column of panels) with the corresponding two-peaked histogram (Fig. 8, cyan boxes). The 30-day orbit would produce too large gaps on the equator for fulfilling the scientific aims of GOCE and is not, therefore, recommended for the measurement phases. A rather small variation in altitude of the repeat orbit with subcycles may therefore produce groundtrack grids, which are not regular after the baseline repeat period is over, corresponding to the repeat orbits of other periods, longer or shorter. This is clearly visualized in the histogram of the equatorial node separation.

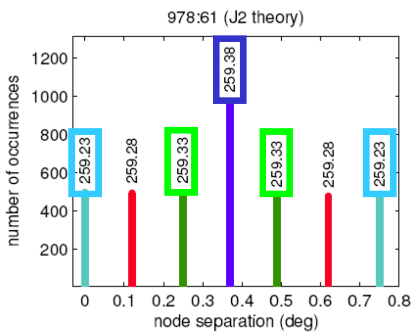

Figure 8. Histogram of the equatorial node separations after the 61-day period has elapsed.

2.3. Analytical vs. numerical modelling of repeat orbit patterns

So far, the graphs shown were calculated using a simple theory with perturbations induced only by the zonal term J2 of the geopotential, corresponding to the Earth flattening. This is reasonable from the point of view of the satellite dynamics, as the J2-induced perturbation is by far the most important and enables one to calculate the satellite orbits with positioning precision of say several hundred metres (for a one-day long integration). What happens to our graphs and conclusions when all other orbital perturbations (full geopotential, lunisolar effects, earth and ocean tides, solar radiation, …) are added? As other small forces act on the satellite as it revolves the Earth, small variations around the J2-induced mean orbit appear. This is shown by the histograms of node separations in Fig. 9. The individual peaks in J2 histograms, corresponding to several 61-day repeat orbits with no subcycles (upper panel), become wider, but still the repeat character of the orbits is kept (lower panel). The graphs showing the global coverage of the Earth by the groundtracks remain almost intact, as the width of the extended peaks of 0.02° in Fig. 9 corresponds to 2 km on the equator, so it is virtually invisible in figures like Fig. 2. Therefore, the simpler J2 theory gives correct general picture of the problem, which is then refined and precisely quantified for drawing the final conclusions by using the full orbital propagator. For the description of the full orbital propagator and perturbations used see [1].

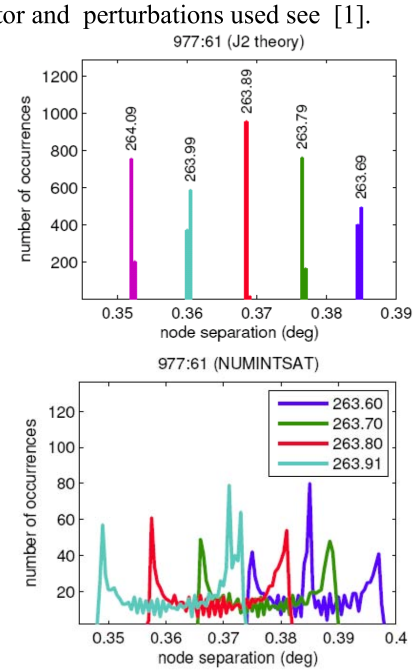

Figure 9. Histograms of the node separation for orbits computed with only J2 perturbation (upper graphs) and using all orbital perturbations (lower graphs).

2.4. Orbits of GOCE near the actual 254.9-km altitude of MOP1

The officially announced orbit intended for the first measurement phase at mean altitude 264 km, a repeat orbit with no subcycles (Fig. 10, orange box), was lowered to 254.9 km due to the unexpectedly low solar activity in 2009. This lower 61-day repeat orbit (blue box) has two subcycles, one about 20 days long, the second with 41-day repeat (red boxes).

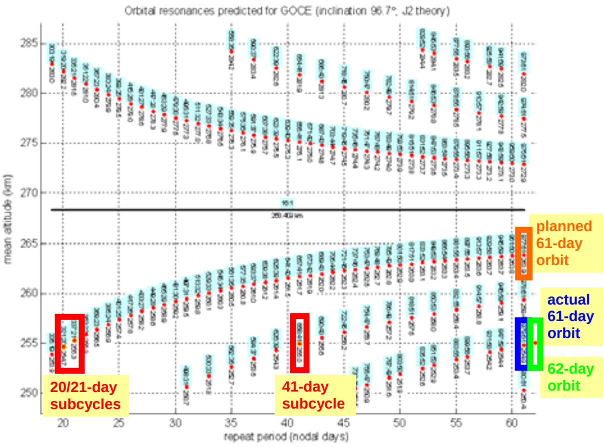

Figure 10. Resonance diagram for GOCE with the MOP1 orbit and its subcycle orbits.

In the following figures we do not show the ground tracks over the whole globe, but just in a narrow zone along the equator. In the upper panels of Fig. 11 we show a portion of the equator with the groundtracks after 65 days; panels differ in the mean satellite altitude (to obtain the graphs, the full orbit propagator was used). The groundtrack grid of the 61-day repeat orbit (in blue) is regular, corresponding to the close double-peak located at the centre of histograms in the lower panel, with the equatorial gaps spread in the interval of 35–45 km. Indeed, in the upper panel of the 61-day orbit, one can see a small irregularity in the longitudinal spacing between the adjacent tracks. In Fig. 11, we also highlighted the 41-day subcycle with the 60-km long equatorial node separations (in red), too long for the required spatial resolution of GOCE. The mean altitude of the 41-day repeat is 100 m above that of the 61-day orbit (see the legend in the lower panel, Fig. 11).

Due to the inclusion of all orbital perturbations, the histogram peaks become wider and more complicated, the points corresponding to repeat orbits in Fig. 10 are somewhat “blurred” and the regular repeat groundtrack grids display small irregularities. Here the fine orbit tuning concept may be applied. Based only on the simple J2 theory, the groundtrack grids of the 61-day and 62-day repeat orbits should be practically the same. But with other perturbations included, the groundtrack grid of the 62-day orbit (Fig. 11, in green) is more regular, its histogram peak being clearly centred at 40 km. Thus by shifting the satellite altitude by 200 m above, one can obtain the most homogeneous coverage of the Earth surface for the two-month measurement period in the considered altitude range.

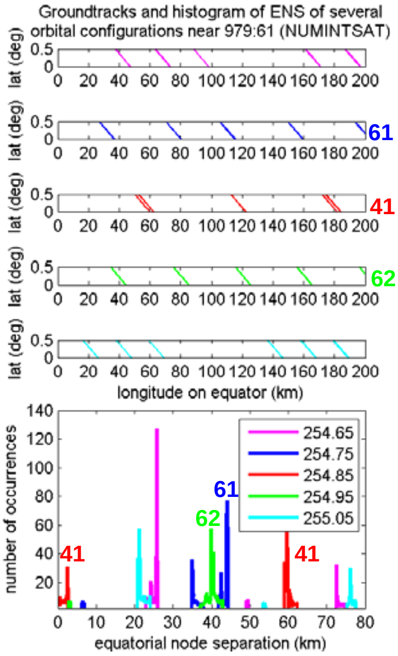

Figure 11. Histograms of node separation for orbits near the 61-day repeat orbit used in MOP1 of GOCE.

2.5. The 145-day repeat orbit with 62-day and 83-day subcycles

At the EGU General Assembly 2010, ESA announced that for the 4th measurement cycle of GOCE a higher, 145-day repeat orbit with a 62-day subcycle is currently being considered [6]. Let us have a closer look at this candidate repeat orbit. The resonance diagram in Fig. 12 shows the proposed 2327:145 orbit at the mean altitude of 255.135 km. After 145 days this orbital configuration would provide a regular grid with the equatorial node spacing of 17.2 km, well enough for the required resolution. In Fig. 12 we have highlighted two repeat orbits, whose altitudes are the closest to the 145-day one: the 62-day orbit, which is lower by 30 m (and was discussed in the previous section), and the 83-day orbit, higher by 23 m (Tab. 1). According to ESA, the GOCE ion thrusters are capable of maintaining the constant altitude within ±50 m [6]. These two near subcycle orbits are therefore at the limit of being distinct from the 145-day orbit. Still, we think that the 145-day orbit is a good choice; even if the altitude control “fails” to keep the constant altitude within say ±20 m during almost 5 months, the node spacings of the two closest subcycle orbits satisfy the required scientific limits, namely, for the 62-day repeat orbit the node spacing is 40.3 km and for 83-day repeat orbit it is 30.1 km.

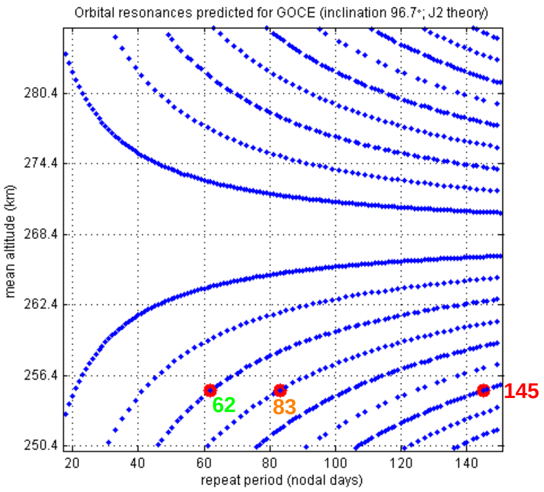

Figure 12. Resonance diagram for GOCE with the 145-day repeat orbit and the two other repeat orbits, which are the nearest in altitude.

Table 1. Resonant orbits ordered by height.

D	R	h (km)
1653	103	255,062
995	**62**	255,105
2327	**145**	255,135
1332	**83**	255,158
1669	104	255,19
2006	125	255,211

3. SUMMARY

Without a "good" coverage even the most sophisticated space instrument (as the GOCE gradiometer certainly is) would not produce "good" geopotential coefficients, i.e. accurate enough up to the given maximum harmonic degree/order. Specifically, we wanted to show that even a small shift in altitude of a few hundred metres may substantially affect the full utilization of the "internal" precision of the instrument by changing the geometry of the ground tracks.

Based on CHAMP and GRACE experience, for GOCE an optimally dense and regular groundtrack grid has been sought by ESA. In Fig. 13, we highlighted orbits (with repeat periods in days), which are very close in their altitude to the proposed 145-day orbit, but each with a very different density of its equatorial node spacing. The highlighted orbits also differ in the regularity of their coverage pattern, when all the satellite perturbations are taken into account, as was explicitly shown on the difference between the 61-day and 62-day repeat orbits. For the success of the GOCE gravity mission, a good choice of the repeat orbit has to be made so that the geometry of the satellite groundtracks enable the full utilization of the fine instrumental accuracy of the first space gradiometer flown by ESA.

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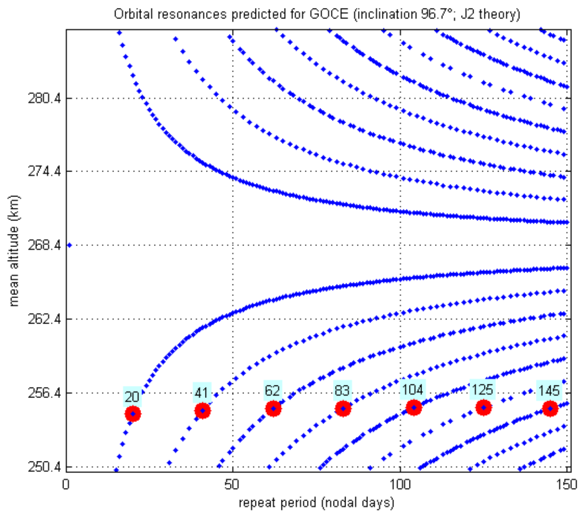

Figure 13. Repeat orbits whose heights are less than 180 m distant from that of the 145-day orbit.

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