Gravity field modelling from kinematic positions using the generalized least squares

Aleš Bezděk [(1)], Jaroslav Klokočník [(1)], Jan Kostelecký[(2)] and Josef Sebera[(1,2)]

[(1)] *Astronomical Institute of the Academy of Sciences of the Czech Republic, 251 65 Ondřejov, Czech Republic, email: bezdek@asu.cas.cz, jklokocn@asu.cas.cz, josef.sebera@fsv.cvut.cz*

[(2)] *Faculty of Civil Engineering, CTU Prague, 166 29 Prague 6, Czech Republic, email: kost@fsv.cvut.cz*

Abstract

The aim of our work is to generate Earth's gravity field models from the GPS positions of CHAMP, GRACE and GOCE, here we will present our method and first numerical results using the real-world data of daily and monthly arcs.

The method is based on Newton's second law of motion, which relates the acceleration of the satellite with the forces acting on it. The acceleration vector of the satellite is obtained through a numerical second-derivative filter applied to kinematic positions, which were calculated from the GPS observations. Forces other than those due to the geopotential are either modelled (lunisolar perturbations, tides) or provided by the onboard measurements (nongravitational perturbations). Then the observation equations are formulated using the gradient of the spherical harmonic expansion of the geopotential and from this linear system the Stokes parameters can be directly obtained.

The problem with the numerical derivative of noisy data (here the GPS positions) is that the second-derivative filter strongly amplifies the noise, especially at high frequencies. Moreover, on combining the data within its window, the digital filter creates an autocorrelation in the observation errors, which further complicates the use of ordinary least squares to obtain the regression solution (e.g. overly optimistic accuracy estimates for a positive autocorrelation).

If errors in a linear regression problem are correlated, i.e. the error covariance matrix is not diagonal, then the so-called *generalized least squares* (GLS) method defines a linear transformation which diagonalizes the error covariance matrix. Subsequently, in the transformed variables the ordinary least squares estimation is used to find the regression parameters with correct estimates of their uncertainties. The GLS estimator is also known as the Aitken estimator and this diagonalization of the error covariance matrix is sometimes mentioned in the formulation of the Gauss-Markov theorem.

In our case, the non-diagonal covariance matrix was generated by the second-derivative filter itself, so finding the GLS transformation matrix is straightforward as the inverse to the second-derivative filter matrix. The application of the GLS transformation is a sort of "double integral", and, effectively, we get back into the positions, but now with the known part of the signal removed. After applying the GLS transformation the errors become again uncorrelated, the variance of noise in the GPS positions may be estimated, and thus the high-frequency noise amplification is eliminated.

We applied the proposed method to CHAMP, GRACE and GOCE data to daily and monthly arcs and successfully obtained the low degree/order part of the geopotential. It is known, and in agreement with our results, that the mean GPS errors are different along the different axes of the satellite local reference frame. We obtained three independent solutions in the along-track, cross-track and radial directions, whose RMS values for the noise in GPS positions of a few centimetres are in accordance with the references. We subsequently combined these solutions using their covariance matrices as weights. Using the full, three-dimensional orbital information is beneficial for the final combined solution. For example, the along-track only solutions have systematically higher error degree variance for certain low degrees – these deficiencies disappear after combining the along-track solution with the cross-track and radial solutions.

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