# GRAVITY FIELD MODELLING FROM KINEMATIC POSITIONS USING THE GENERALIZED LEAST SQUARES

**Aleš Bezděk[1]*, Jaroslav Klokočník[1], Jan Kostelecký[2], Josef Sebera[1,2]**

[1]Astronomical Institute, Academy of Sciences of the Czech Republic (*bezdek@asu.cas.cz)
[2]Czech Technical University in Prague

## General characteristics of the method used

- Satellite acceleration approach based on Newton second law used as vector equation
- No a priori gravity model needed, Stokes parameters (SP) obtained directly through linear regression
- Three independent solutions [along-track (A-T), cross-track (C-T), radial (RAD)] can be optimally combined using normal matrices → deficiencies of along-track solution are mitigated
- Problems of noise amplification and autocorrelation are avoided using the generalized least squares method
- Stokes parameters were successfully obtained from real-world orbits of GRACE (years) and GOCE (months)

## Method of inversion

### (1) GPS-based accelerations

- Kinematic positions **r** → second-derivative digital filter F
  → GPS-based (observed) accelerations $d^2\mathbf{r}/dt^2 \approx \mathbf{a}_{GPS}(\mathbf{r}) \equiv F * r$

### (2) Newton second law

- $\mathbf{a}_{GPS} \equiv d^2\mathbf{r}/dt^2 = \mathbf{a}_{geop} + \mathbf{a}_{LS} + \mathbf{a}_{TID} + \mathbf{a}_{NG}$
  where the accelerations are due to:

$\mathbf{a}_{geop}(\mathbf{r}) \equiv \sum SP \times \nabla SSH(r,\theta,\varphi)$ geopotential in sph. harmonics SSH

$\mathbf{a}_{LS}$ … lunisolar effects

$\mathbf{a}_{TID}$ … solid Earth and ocean tides

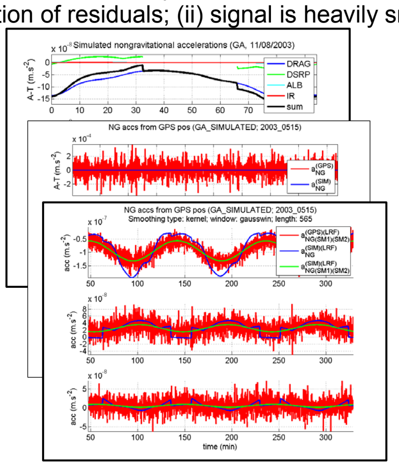
$\mathbf{a}_{NG}$ … acc. of nongravitational origin (drag, radiation pressures)

### (3) Stokes parameters (SP)

- Obtained by means of linear regression from linear system:
  $\sum SP \times \nabla SSH(r,\theta,\varphi) = \mathbf{a}_{GPS} - (\mathbf{a}_{LS} + \mathbf{a}_{TID} + \mathbf{a}_{NG})$

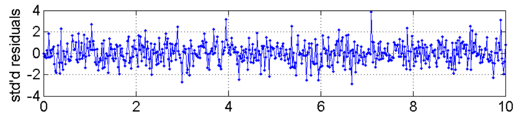
### (4) Problem of amplified noise

- Second derivative filter F in **(1)** amplifies the noise in the GPS positions, especially high-frequency noise
- Figs: example for nongravitational forces, the "true" signal $\mathbf{a}_{NG}^{(SIM)}$ is buried in noise
- Moreover, through the action of filter **F**, the noise becomes also autocorrelated (more about it in **(5)**)
- Smoothing is a possible way out, but it creates: (i) further autocorrelation of residuals; (ii) signal is heavily smoothed too

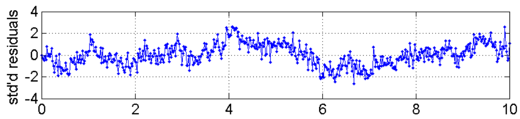



### (5) Problem of autocorrelated noise

- Ordinary least squares (OLS) provide correct uncertainty estimates for the fitted parameters (the usual "3-σ rule"), if the errors are independent and normally distributed
- If random errors are positively correlated (so called "problem of autocorrelated errors" in OLS), then:
  - uncertainty (error bar) of fitted parameters is underestimated
  - one gets overly optimistic accuracy estimates
- In **(3)** filter F generates a correlation structure of the random noise in $a_{GPS}$
- Example: White noise




- Example: Autocorrelated random noise




### (6) Generalized least squares (GLS)

- If errors in an OLS problem are correlated → the covariance matrix is not diagonal, $C \neq \sigma^2 \boldsymbol{I}$
- Then the GLS defines linear transformation W such that the new covariance matrix is diagonal. The transformation matrix $W = T^{-1}$, where $C = TT'$ (T is sometimes called the "square root" of C)
- In the transformed variables, the usual OLS is used to find the regression parameters with correct estimates of their uncertainties
- GLS is a linear transformation of Eq. **(3)** to decorrelate the covariance matrix of observations: this is sometimes already "contained" in the formulation of the Gauss-Markov theorem, which states that the OLS estimator is BLUE estimator

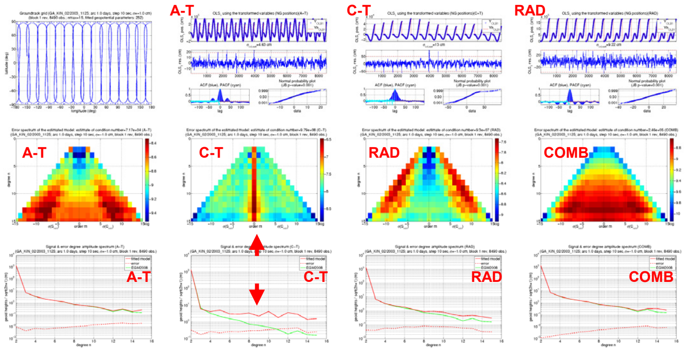
### (7) Use of GLS to remove autocorrelation and HF noise

- In fact, the non-diagonal covariance matrix was generated by action of the second derivative filter F: $C = \sigma^2 FF'$, where $\sigma^2$ is the variance of the white noise in the GPS positions → finding the GLS transformation matrix is straightforward: $W = F^{-1}$
- Since filter F is FIR filter (finite impulse response), the discrete convolution with it may be formulated using a matrix; we compute W as the inverse of the corresponding filter matrix
- $W = F^{-1}$ is an inverse to the second derivative filter → application of transformation W is thus a sort of "double integral"
- Effectively, we got back into the positions, but now with the known part of the signal removed
- After applying the GLS transformation matrix W to **(3)**:
  - Residuals become again uncorrelated
  - Variance $\sigma^2$ of noise in GPS positions may be estimated
  - High-frequency noise amplification is avoided

## Results for real orbits

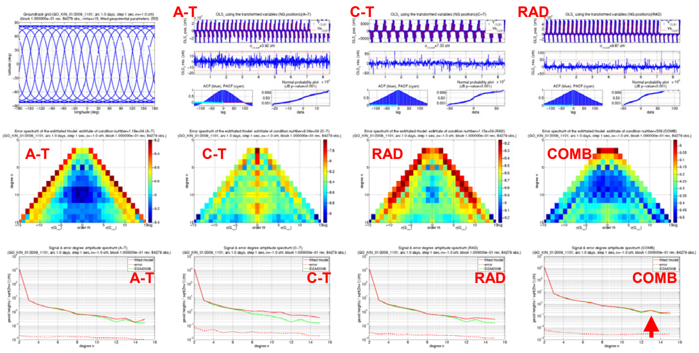
### (8) GLS inversion: GRACE  A (1 day; max. degree 15)

- Vector eq. **(3)** → three independent solutions (A-T, C-T, RAD)
- Combination solution (COMB) obtained using normal matrices
- Polar orbit → low sensitivity of C-T solution to zonal harmonics



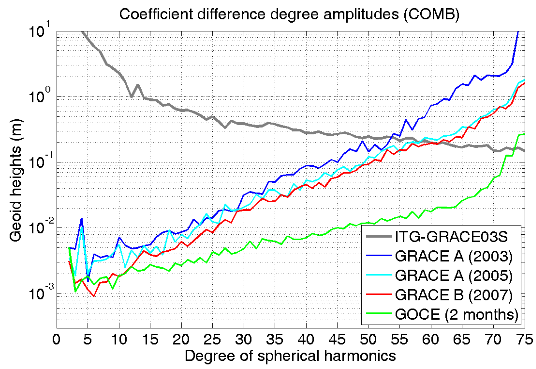

### (9) GLS inversion: GOCE  (1 day; max. degree 15)

- Better results at higher degrees due to lower altitude



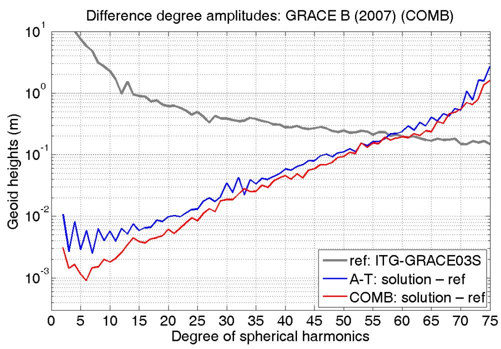

### (10) Long-term solutions

- Shown are differences to a superior model ITG-GRACE03S
- Yearly GRACE solutions better at lower degrees
- Monthly GOCE solutions better at higher degrees (11–12/2009)



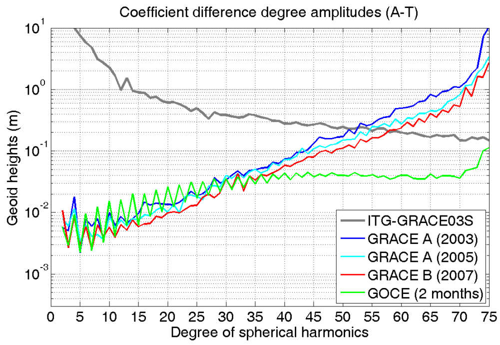

### (11) Along-track vs. combined solution

- Systematically, A-T solutions have worse results for certain low degrees (e.g. 4), similarly to energy-balance-based results [2]
- Adding C-T and RAD solutions to A-T → COMB solution is OK



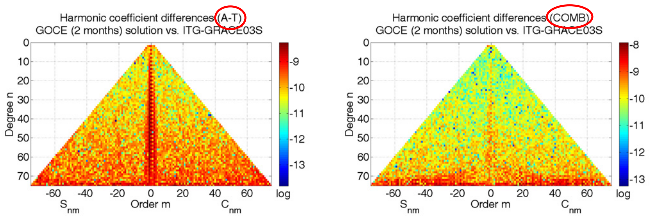

### (12) Along-track vs. combined solution

- Compared with the reference model, A-T solutions give systematically worse results than COMB solutions
- Figure: Degree differences of A-T solutions to be compared with those of COMB solutions in Fig. **(10)**



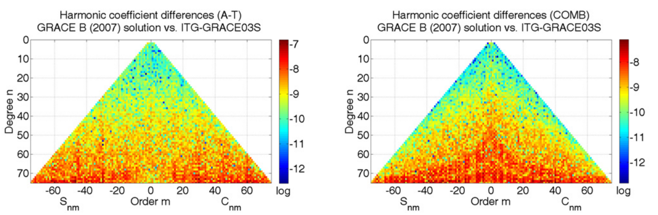

### (13) Zonal coefficients – GOCE

- Polar caps not being sampled (inclination 96.6°) → worse zonal and near-zonal coefficients esp. in A-T solutions (left panel)
- This unwanted effect is almost disappearing in COMB solutions!




### (14) Zonal coefficients – GRACE

- Small polar gap (inclination 89.0°), no deterioration of zonals




## Conclusions

- Properties of the obtained geopotential solutions compare well with those of other research groups (e.g. [3])
- Monthly and yearly solutions show clearly that it is a good idea not to lose the three-component orbital information and that the combined solutions are more precise compared to those based only on the along-track direction, and, in particular, the combined solutions exhibit no worsening of low zonal and near zonal coefficients, even for GOCE


### (15) Acknowledgments

Our thanks are due to Dražen Švehla, Adrian Jäggi and Markus Heinze for providing kinematic orbits of GRACE satellites, to ESA for orbital data of GOCE, to ICGEM/GFZ for geopotential models. This work has been supported by the ESA/PECS grant project No. 98056.


### (16) References


[1] Bezděk A, Klokočník J, Kostelecký J, Floberghagen R, Sebera J, 2010. Fine orbit tuning to increase the accuracy of the gravity-field modelling. AGU Fall Meeting, 13–17 Dec., San Francisco, California, USA.

[2] Weigelt, M., Sideris, M. G., Sneeuw, N., 2009. On the influence of the ground track on the gravity field recovery from high-low satellite-to-satellite tracking missions: CHAMP monthly gravity field recovery using the energy balance approach revisited. J Geod 83, 1131–1143.

[3] Jäggi, A., Bock, H., Prange, L., Meyer, U., Beutler, G., 2011. GPS-only gravity field recovery with GOCE, CHAMP, and GRACE. Adv. Sp. Res. 47, 1020–1028



**Presented at 4th International GOCE User Workshop, Technische Universität München, Germany, 31 March – 1 April 2011**