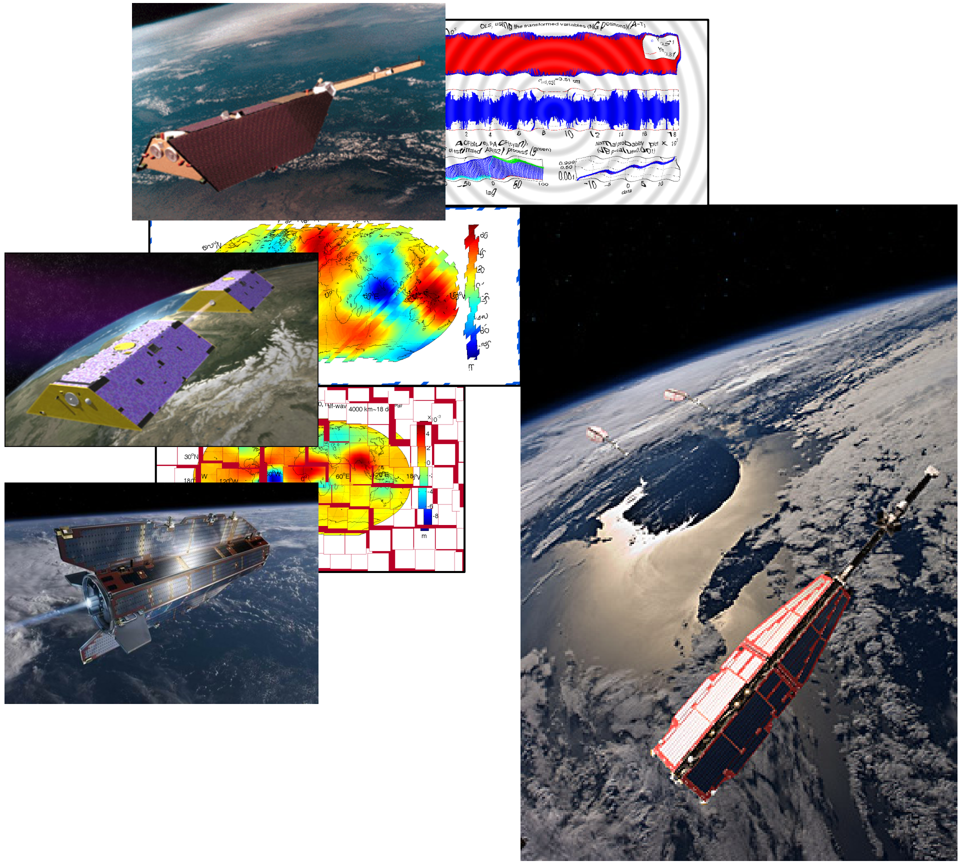
Calibration of Swarm accelerometers by means of kinematic orbits and gravity field models



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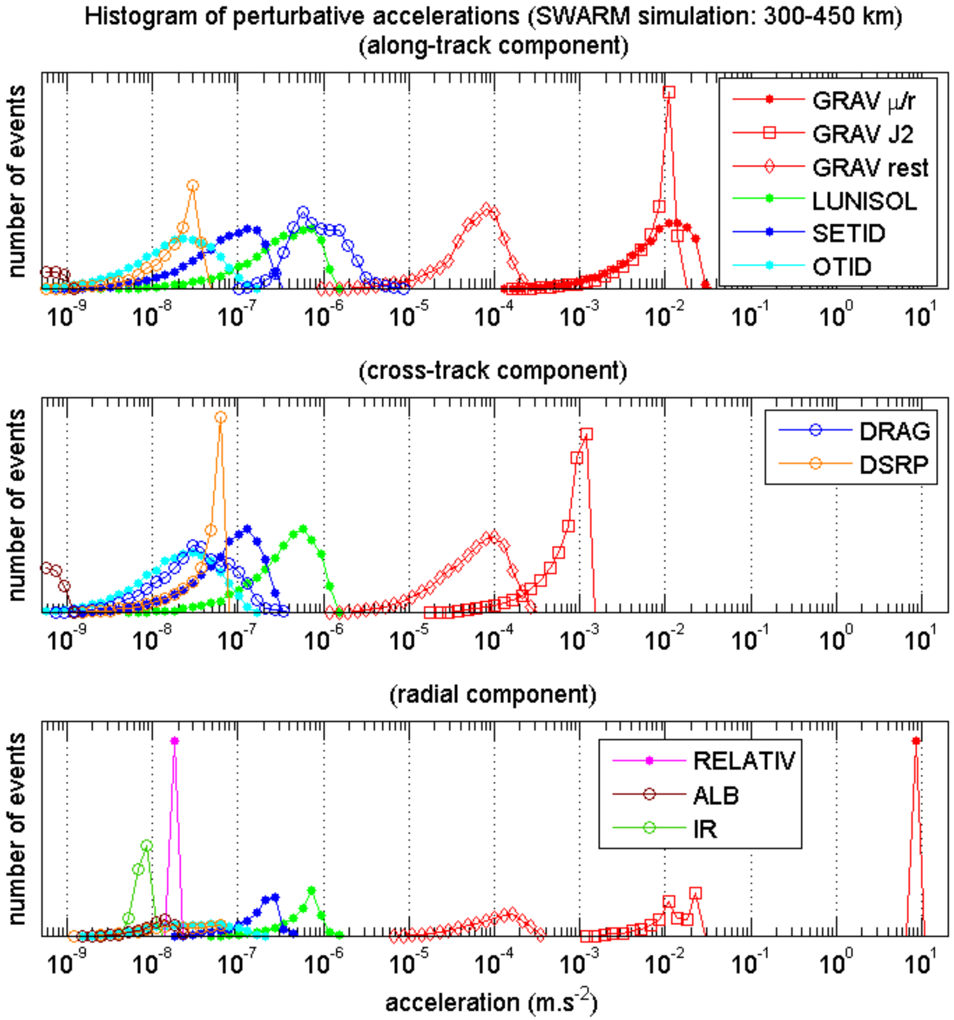
First Swarm Validation Workshop,
ESA/ESRIN Frascati, 20 June 2012

Calibration of Swarm accelerometers by means of kinematic orbits and gravity field models

- Background and objectives
- Calibration method
- Results using real data
- Contribution to SWARM validation activities

Forces acting on LEO satellites

- Satellites in **Low Earth Orbits** (LEO): altitudes 100-2000 km
- Dominant is the central gravitational field
- Other forces act as small perturbations



Accelerations of gravitational origin

- central geopotential term
- noncentral geopotential terms
- lunisolar perturbations
- solid-Earth and ocean tides
- relativistic effects

Nongravitational accelerations

- atmospheric drag
- radiation pressures
  (direct solar, albedo, terrestrial infrared)

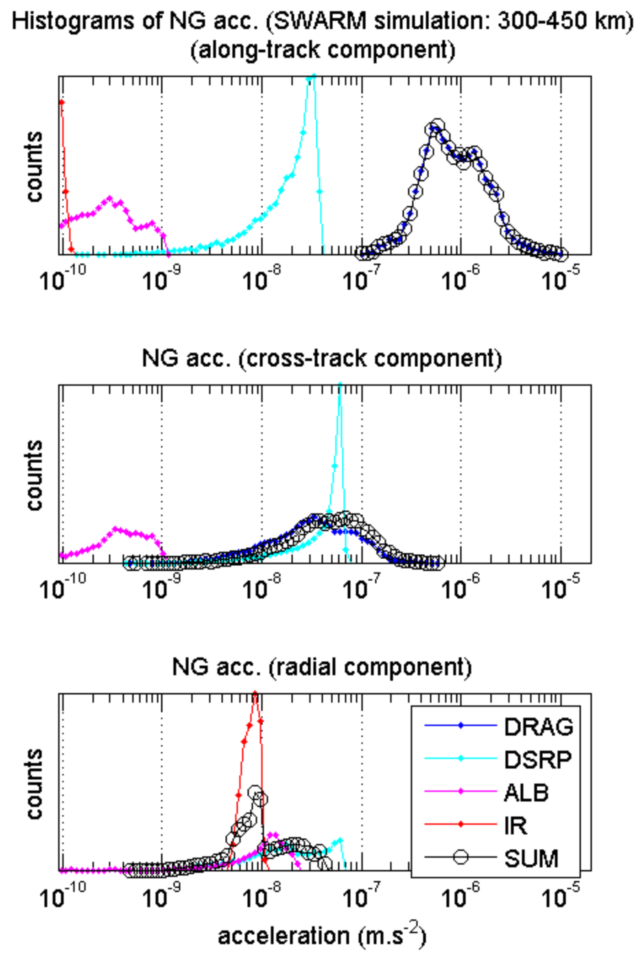
- **Magnitudes**
  $\mu/r \gg J2 \gg$ GRAV rest, DRAG, LUNISOL > SETID > OTID,DSRP > ALB, IR, RELAT

Space accelerometers

- Designed to measure tiny nongravitational accelerations
- Aboard the satellites: CHAMP, GRACE, GOCE

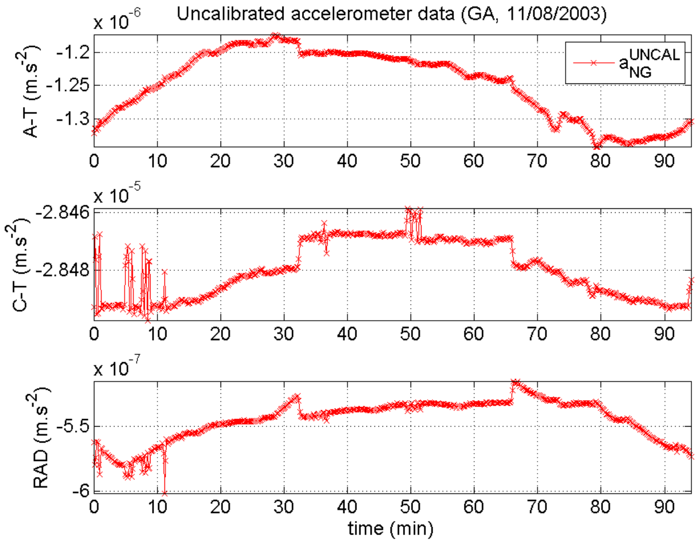
- Magnitude of nongravitational accelerations:
  - Along-track: $10^{-7} - 10^{-5}$ m.s^{2}
  - Cross-track: $10^{-9} - 10^{-6}$ m.s^{2}
  - Radial: $10^{-9} - 10^{-7}$ m.s^{2}

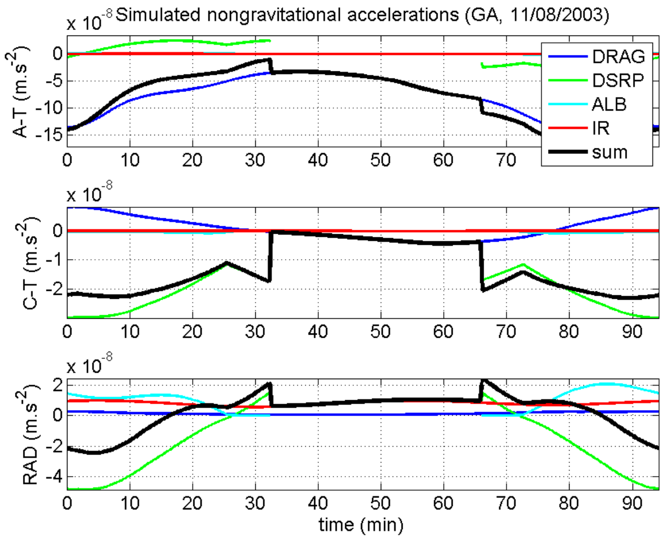
- Due to the smallness of the nongravitational signal compared to gravity, **space accelerometers cannot be calibrated on the ground**.



Uncalibrated ACC data

- **Waveforms** of uncalibrated ACC signal and simulated NG accelerations are quite **similar**.
- **Spikes** in cross-track and radial components correspond to cold-gas **thruster firings**.
- RAD component
  - ➢Orbit geometry → passage through zero
  - ➢ACC data have **offset**
- Comparison of range on y-axes
  - ➢ACC data are orders of magnitude out
- **Need to calibrate the ACC readouts**





Acceleration approach

- GPS positions **r** with constant time step → **numerical second derivative** of **r**(t) → **GPS-based accelerations** $d^2\mathbf{r}/dt^2 \approx \mathbf{a}^{(GPS)}$

Newton's second law: $\mathbf{a}^{(GPS)} \approx d^2\mathbf{r}/dt^2 = \mathbf{a}_{geop} + \mathbf{a}_{LS} + \mathbf{a}_{TID} + \mathbf{a}_{REL} + \mathbf{a}_{NG}$

$\mathbf{a}_{geop}(\mathbf{r}) \equiv \sum GC\times \nabla SSH(r,\theta,\varphi)$ … geopotential in spherical harmonics

$\mathbf{a}_{LS}$, $\mathbf{a}_{TID}$, $\mathbf{a}_{REL}$, $\mathbf{a}_{NG}$ … lunisolar action, tides, relativity, nongravitational forces

Two applications:

1) Assume the geopotential is known and define GPS-based NG accelerations

$$\mathbf{a}_{NG}^{(GPS)} = \mathbf{a}^{(GPS)} - (\mathbf{a}_{geop} + \mathbf{a}_{LS} + \mathbf{a}_{TID} + \mathbf{a}_{REL})$$

$$\mathbf{a}_{NG}^{(GPS)} = \mathbf{B} + \mathbf{S}.\mathbf{a}_{ACC}^{(UNCAL)} + \boldsymbol{\varepsilon} \qquad (*)$$

Calibration parameters B/S for ACC are obtained by solving linear system (∗).

2) On rearranging the observation equations:

$$\sum \mathbf{GC}\times\nabla SSH(r,\theta,\varphi) + \boldsymbol{\varepsilon} = \mathbf{a}^{(GPS)} - (\mathbf{a}_{LS} + \mathbf{a}_{TID} + \mathbf{a}_{REL} + \mathbf{a}_{NG})$$

Now **Geopotential coefficients GC** can be solved for.

ACC calibration by acceleration approach: ASU[1] version

Linear system of observation equations to estimate **calibration parameters B/S**:

$$\mathbf{a}_{NG}^{(GPS)} = \mathbf{B} + \mathbf{S}.\mathbf{a}_{ACC}^{(UNCAL)} + \boldsymbol{\varepsilon} \qquad (*)$$

- **Calibration standard**: GPS-based NG accelerations, $\mathbf{a}_{NG}^{(GPS)} = \mathbf{a}^{(GPS)} - \mathbf{a}_{GRAV}$
- Assumption: uncertainty in modelled accelerations of gravitational origin

  $\mathbf{a}_{GRAV} = \mathbf{a}_{geop} + \mathbf{a}_{LS} + \mathbf{a}_{TID} + \mathbf{a}_{REL}$ is negligible relative to that of $\mathbf{a}^{(GPS)}$

- Problem: **Numerical derivative amplifies noise** in GPS positions
  - Solution: **Generalized least squares (GLS)**

    → linear transformation of system (∗)

- Problem: Real data → GPS positions have **correlated errors**
  - Solution: **partial autocorrelation function (PACF)** → **autoregressive model (AR)**

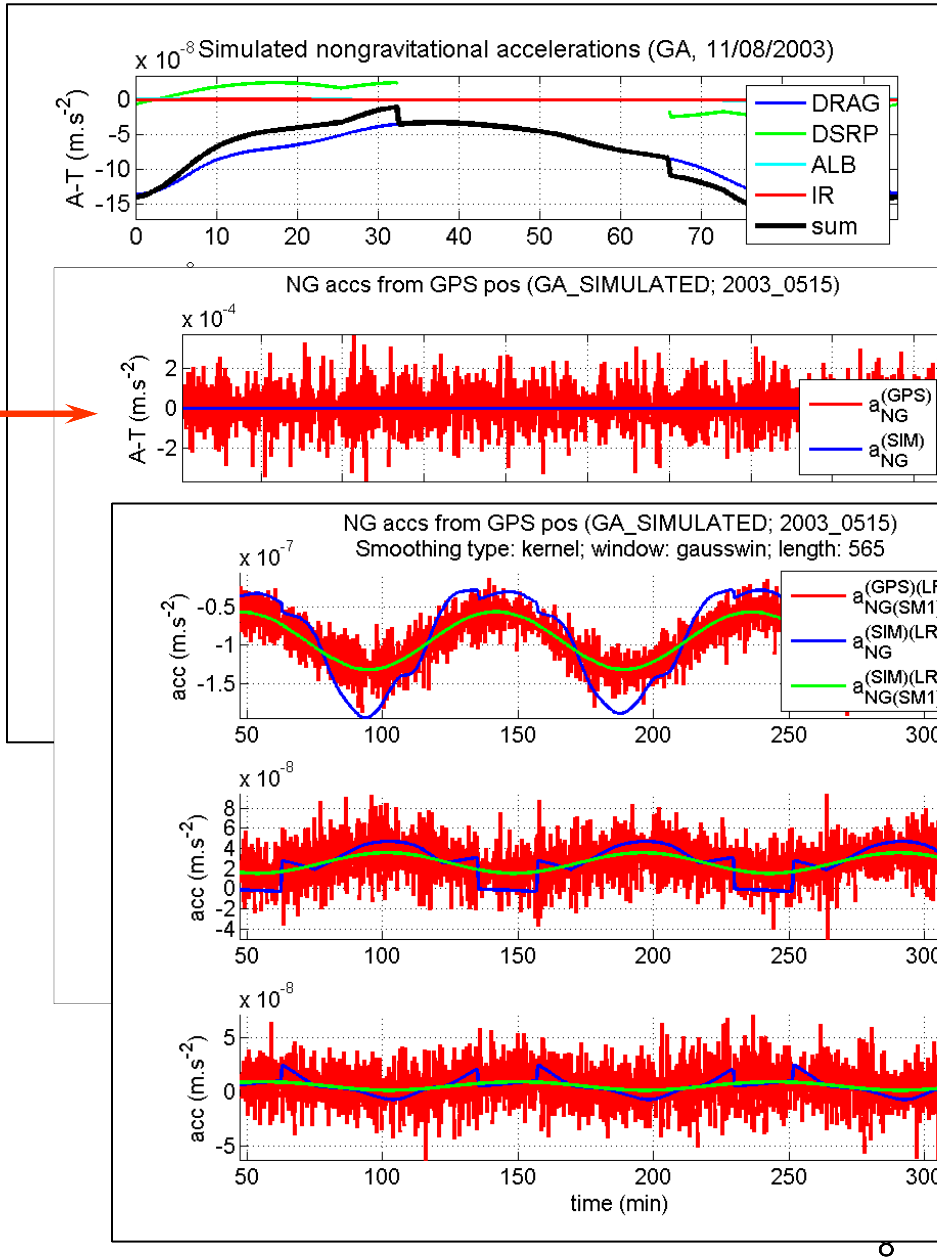
    → linear transformation of system (∗)

- Solving transformed system (∗) we get calibration parameters **B/S** by ordinary least squares.

[1]*ASU…Astronomical Institute ASCR*

Problem of amplified noise

- Accelerations: $d^2\mathbf{r}/dt^2 \approx \mathbf{a}^{(GPS)} \equiv F * r$
- The **second derivative filter F** amplifies the noise in the GPS positions, especially **high-frequency noise**
- Figs: example for nongravitational forces, the "true" signal $\mathbf{a}^{(SIM)}_{NG}$ is buried in noise
- Moreover, through the action of filter **F**, the noise become also **autocorrelated**
- **Smoothing** is a possible way out, but:
  - further autocorrelation of residuals
  - signal is heavily smoothed too



Problem of autocorrelated noise

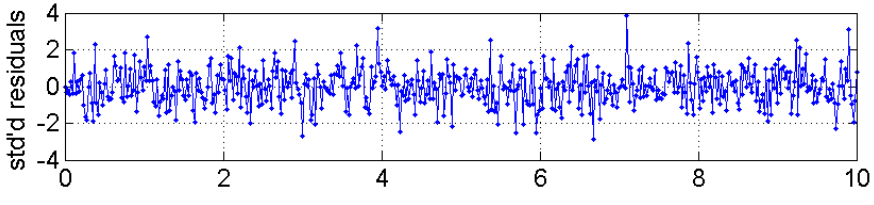
Linear system of observation equations to estimate calibration parameters B/S:

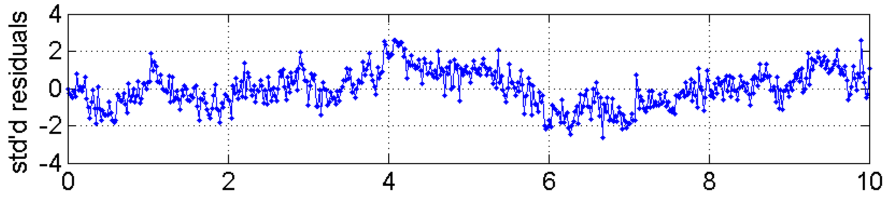
$$\mathbf{a}_{NG}^{(GPS)} = \mathbf{B} + \mathbf{S}.\mathbf{a}_{ACC}^{(UNCAL)} + \boldsymbol{\varepsilon} \qquad (*)$$

- **Ordinary least squares** (OLS) provide **correct** uncertainty estimates for the fitted parameters (usual ”3-σ rule”), if the errors $\boldsymbol{\varepsilon}$ are **independent** and **normally** distributed

- If the random **errors** are **positively correlated**
  - ➢ **uncertainty** in the fitted parameters is **underestimated**
  - ➢ overly optimistic accuracy estimates

- In Eq. ($*$) filter **F** generates a correlation structure of random noise in $\mathbf{a}_{NG}^{(GPS)}$





Generalized least squares (GLS)

- Autocorrelated errors in OLS problem → non-diagonal covariance matrix $C \neq \sigma^2 \boldsymbol{I}$

- **GLS** defines **linear transformation**: $W=T^{-1}$, where $C=TT'$ (T … "square root" of C)
  → new covariance matrix is diagonal

- In transformed variables, OLS may now used
  - correct estimates of mean values and uncertainties of the parameters
  - correct estimates of confidence intervals

- In statistics, GLS estimator is also called the **Aitken estimator**

- Sometimes the GLS method is already "contained" in **Gauss-Markov theorem** when using weight matrix $P=C^{-1}$

Use of GLS to remove autocorrelation and HF noise

Linear system of observation equations to estimate calibration parameters B/S:

$$\mathbf{a}_{NG}^{(GPS)} = \mathbf{B} + \mathbf{S}.\mathbf{a}_{ACC}^{(UNCAL)} + \boldsymbol{\varepsilon} \qquad (*)$$

- In fact, **non-diagonal covariance** matrix was **generated by** the second derivative **filter** F: $C=\sigma^2FF'$, where σ^2 is the variance of the white noise in GPS positions

  → finding the **GLS transformation matrix** is straightforward: $\mathbf{W=F^{-1}}$

- $W=F^{-1}$ is inverse to second derivative filter → application of W is "double integral"
- Effectively, we got back into the positions, but now with known part of signal removed

- After applying the GLS transformation W to eq. (∗):
  - the residuals become again uncorrelated
  - the variance σ^2 of **noise in GPS positions** may be **estimated**
  - high-frequency **noise amplification** is **eliminated**

Decorrelation of GPS position errors using AR process

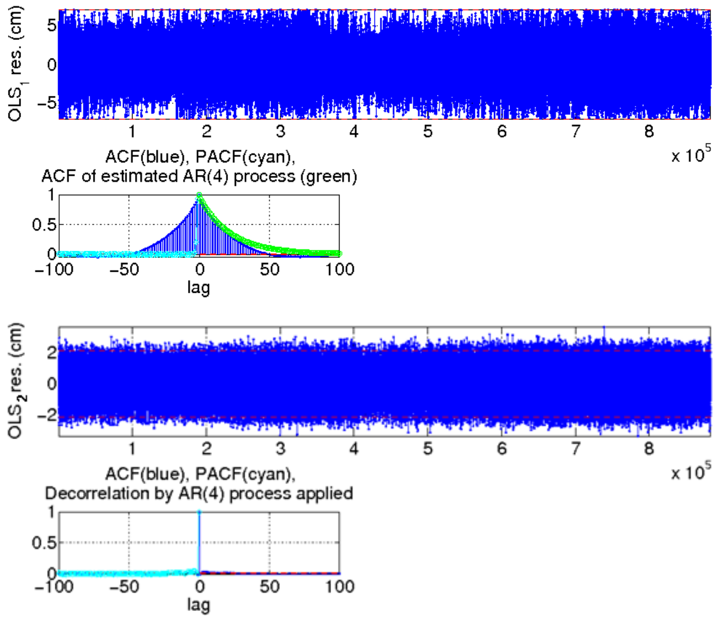
Problem: Real GPS positions have correlated errors

- Indicated by sample autocorrelation function ACF

Partial autocorrelation function PACF

Rapid decay of PACF → suitability of AR model to represent the correlation structure

- In figure, fitted **autoregressive model AR** of order 4 approximates ACF of residuals



Decorrelation of residuals using fitted AR models

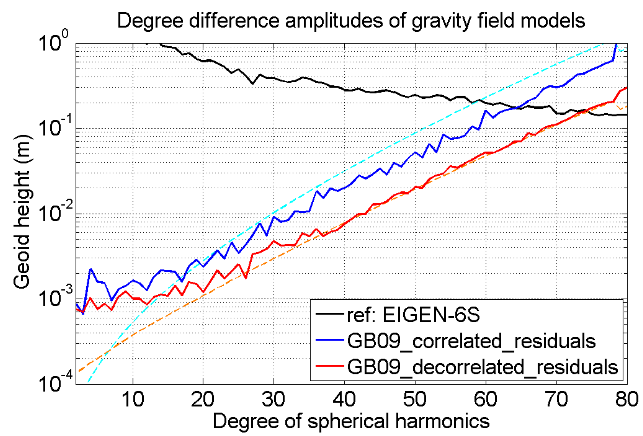
- by linear transformation of calibration system (∗)
- ACF and PACF become approx. delta functions

Calibration parameters B/S have **realistic error bars**

- **Correct** estimates of **ACC data uncertainty!**

Estimation of **geopotential coefficients GC**

- After decorrelation, GC are **more accurate** by factor 2–3!
- More realistic uncertainty estimate of GC



Real data: calibration of GRACE accelerometers

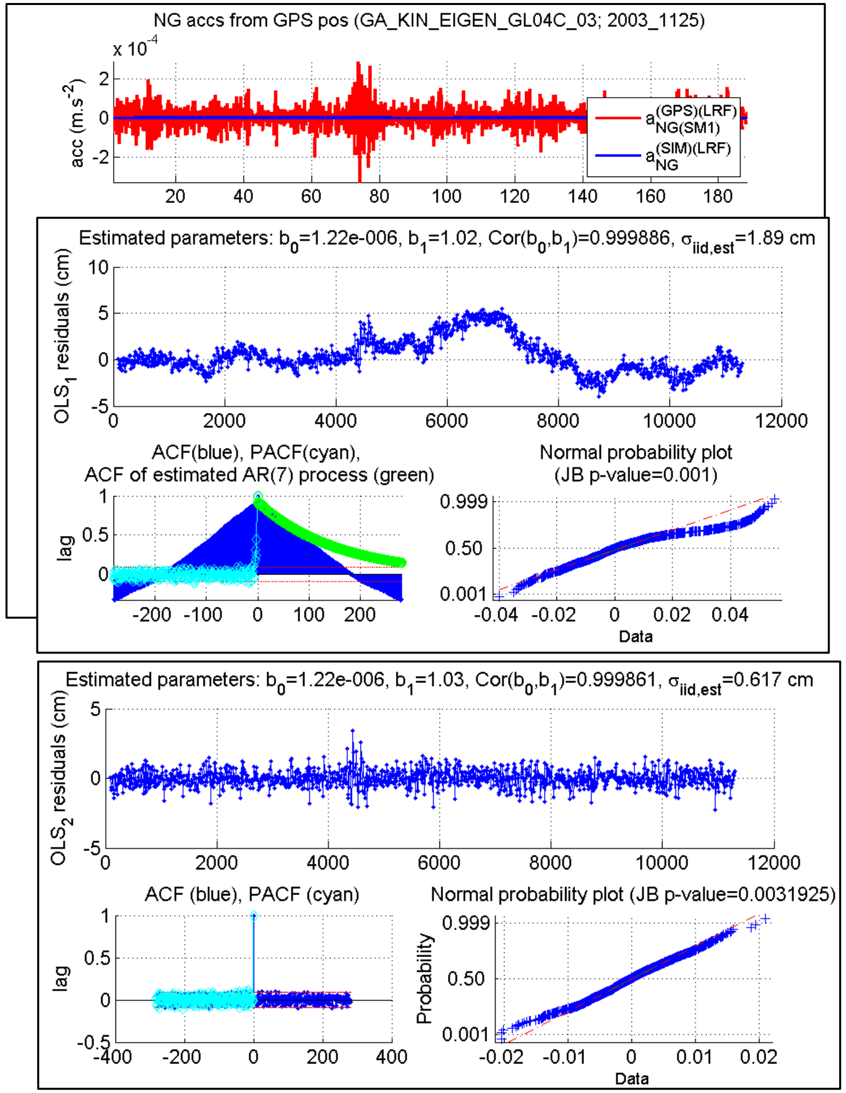
- Period of two satellite revolutions
- Amplification of noise due to second derivative

- Residuals after first linear transformations
- Errors in GPS positions of few cm

- Obvious autocorrelation (sample ACF)
- PACF falls down quickly
  → we estimated AR process of order 7

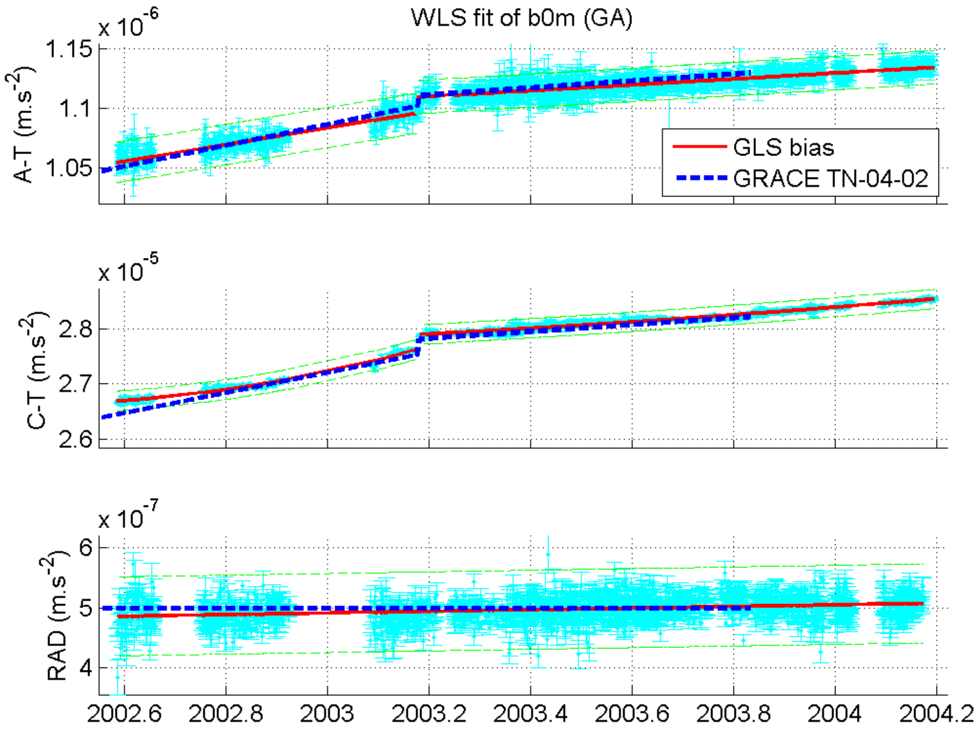
- Residuals after second linear transformation derived from fitted AR(7) process

- Both ACF and PACF now indicate that residuals become uncorrelated



Calibration of 1.5 years of GRACE accelerometers data

- Due to a large bias B, the calibration parameters B and S are highly correlated
- We set S fixed after GRACE TN-04-02 → bias offsets have similar time evolution



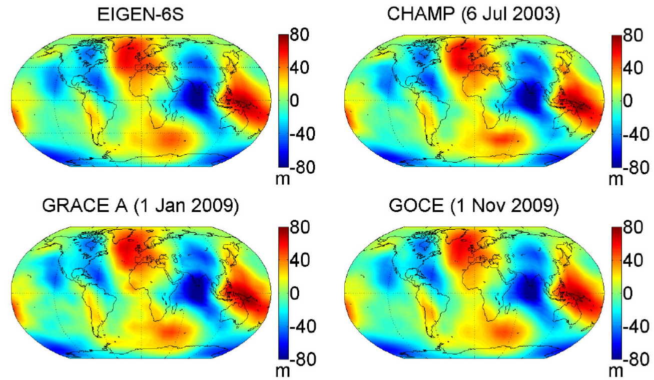
Further information

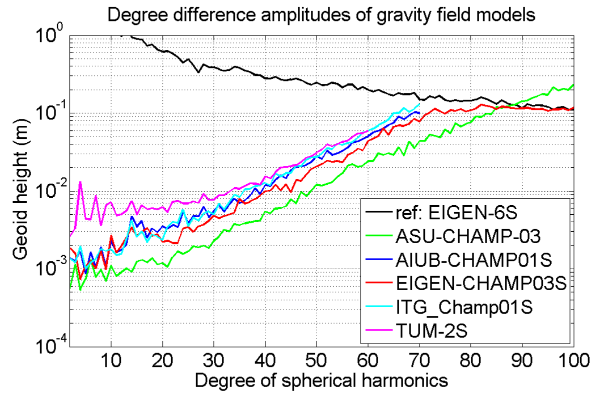
- http://www.asu.cas.cz/~bezdek/vyzkum/
- Bezděk A, 2010. Calibration of accelerometers aboard GRACE satellites by comparison with POD-based nongravitational accelerations. *Journal of Geodynamics* 50, 410–423.

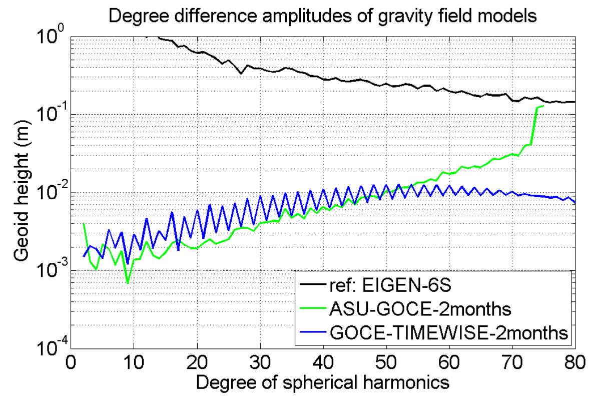
Gravity field models from orbits of CHAMP, GRACE, GOCE

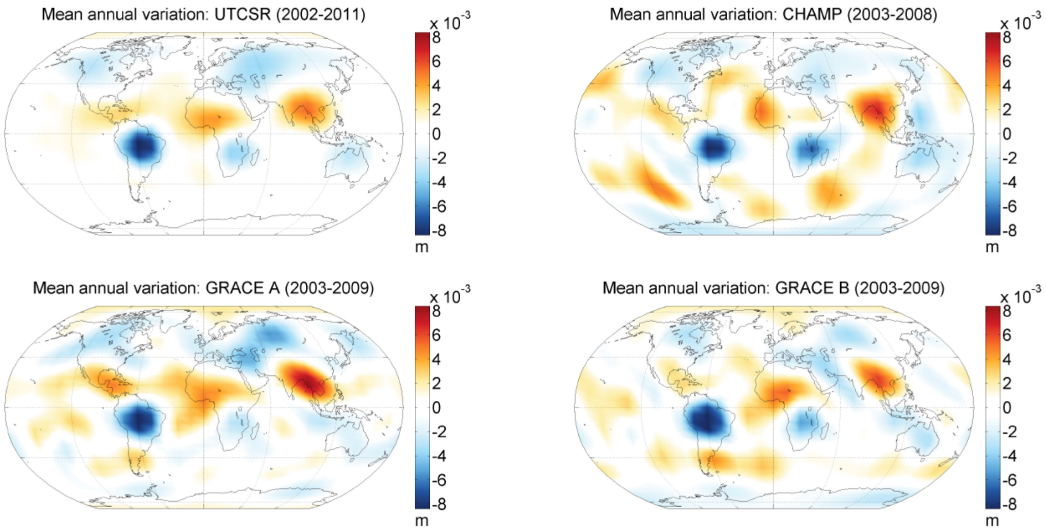
Examples of successful application of the presented calibration method to estimate geopotential coefficients.

- One-day solutions
- CHAMP yearly solution for 2003
- GOCE two-month solution
- Average hydrological signal in CHAMP and GRACE solutions over 7 years (time-variable gravity)









Contribution to SWARM validation activities

Pertinent Level-2 products

- ACCxCAL_2_: Accelerometer calibration parameters for all three SWARM satellites
- ACCx_AE_2_: Time series of calibrated accelerometer data
- Level 2 precise kinematic orbits

The presented ACC calibration method

- Independent assessment of ACC calibration parameters
- Output: calibrated ACC measurements together with realistic uncertainty estimates
- Implemented solely by our team, independent of any other orbital sw package
- Was tested using real-world ACC data of CHAMP and GRACE with positive results
- No special manoeuvres are necessary, method is applicable during science free-fall regime
- During routine phase: it may operate offline (CAT-1 algorithm)

Apart from calibrating ACC data, we will compute **SWARM-only gravity field models**

Thank you for your attention