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Orbital elements of multiple spectroscopic stars

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Abstract. — A method for simultaneous decomposition of spectra of binary and multiple stars and the solution of orbital elements is described.

Key words: stars: binaries spectroscopic — methods: numerical

1. Introduction

Since the discovery of binary stars, systematic observations of binaries and multiple stars and determination of their orbits has played a crucial role in quantitative testing of theories of stellar structure and evolution. The classical methods of observation of visual, eclipsing and spectroscopic binaries have been complemented by methods of interferometry, polarimetry and detailed analysis of stellar spectra. However, because the range of applicability of each method is partly different, it is useful to combine them in order to achieve consistent and complex information on systems under study.

In view of this basic philosophy, the code FOTEL was developed by the author at Ondřejov observatory to solve light curves (i.e. to get '**phot**ometric **el**ements' including the orbital ones) using the complementary information contained in radial-velocity curves (Hadrava 1990; for detailed information see the manual accessible together with the code by FTP at directory `/ftp/pub/fotel` at `SUNSTEL.ASU.CAS.CZ`). The release 2 of the code is based on a relatively simple physical model to enable handling with extensive data files. Owing to its flexibility, it proved to be a useful tool even for pure radial-velocity data, despite its main purpose – the solution of light curves – makes the code more complicated than it could be for simple solution of RV-curves (see e.g. Mayer et al. 1991).

One general problem of the solution of orbital elements via the radial-velocity curves is that the lines corresponding to the component stars must be well resolved. For binaries with two spectra visible, consisting of stars of close spectral types (and comparable magnitudes), this is possible only in extremes of radial velocities, while the data points close to the system velocity are unreliable due to the blending of the lines. Just these points can be important for determination of the eccentricity and periastron longitude. Systematic errors can appear in binaries with only one spectrum distinguished if lines of the secondary star are hidden in the line profiles of the primary. The situation is even worse if also lines of a third star are present. Moreover, the spectral resolution of composite spectra depends on the dispersion of the spectrograph, on the quality of the detector and signal to noise of each exposure. These problems are more pronounced in early type stars whose line profiles are usually broadened by rapid rotation (Schönberner & Harmanec 1995).

It is clear that even in the case of blended lines information on the radial velocities of the component stars is contained in the line profile. To get this information it is possible to use the cross-correlation technique comparing the observed spectrum with masks similar to expected spectra of the components. These can be obtained either from reference stars of similar types or from model spectra. The value of this approach is well known from the fine results achieved (see e.g. Simkin 1973; Hill 1993). However, to choose an appropriate mask can be a problem if the lines of two or more stars are blended in all phases. On the other hand, it is clear that the best mask would be the spectrum of the star itself (provided it does not change with the phase) and it should be possible to obtain it comparing profiles in different phases.

A method comparing the profiles at different phases is the 'disentangling' by singular value decomposition (Simon & Sturm 1994), which solves for the spectra of components from a huge sparse system of linear equations with several observed spectra on the right hand side.

An alternative method of disentangling which employs the Fourier transform (like in the correlation technique) is presented in this paper. This method can be used as the first step in the standard procedure of determination of radial velocities from each exposure. A by-product of this data processing are the mean (from all exposures) spectra of individual binary components, which can be used

for a detailed spectroscopic analysis. An advantage of this method is that all lines in the spectra are used for radial-velocity determination even if their identification is not clear (certainly, this can turn into disadvantage if variable features which do not follow the velocity of the star are present in the spectra). The velocities obtained in the first step can then be fitted by radial-velocity curves in a classical way (e.g. by FOTEL). However, in the 'pure' cases when the radial velocities should be given by a known law e.g. of orbital motion (i.e. if there is no distortion due to circumstellar matter or other effects) the intermediate stage of determination of independent radial velocities from individual exposures complicates the task of spectra disentangling. For these cases the method can fit directly the orbital elements from the observed spectra and to yield the mean component spectra with a smaller number of free parameters.

The mathematical principle of the method described in Sect. 2 is based on the least-squares fitting of transformed spectra combining direct analytical calculation of linear coefficients with optimization in nonlinear ones. This combined optimization (also used in FOTEL) of component spectra and **or**bital **el**ements is performed in the code KOREL which is complementary to FOTEL and uses similar inputs – see Sect. 3. An example and concluding remarks are given in Sect. 4. A detailed analysis of real data will be presented in subsequent papers.

2. Disentangling of composite spectra through the Fourier transform

Let the observed spectrum $I(x,t)$ be composed of spectra $I_j(x)|_{j=1}^n$ of n stars, each one Doppler shifted according to the instantaneous radial velocity $v_j(t)$ of the star j at the time t. Suppose the spectra of individual stars have no intrinsic variability. If the variable x is chosen to be logarithmic in wavelength, $x = c\ln(\lambda)$, then (for $v_j(t) \ll c$) the composed spectrum is given by convolution in x

$$I(x,t) = \sum_{j=1}^{n} I_j(x) * \delta(x - v_j(t)) \ . \tag{1}$$

The Fourier transform ($x \rightarrow y$) of this equation reads

$$\tilde{I}(y,t) = \sum_{j=1}^{n} \tilde{I}_j(y) \exp(iyv_j(t)) \ . \tag{2}$$

If we have k spectra ($k > n$) observed at $t_l|_{l=1}^k$ corresponding to various values of $v_j(t_l)$, we can – in principle – fit them searching for appropriate values of $v_j(t_l)$ and $\tilde{I}_j(y)$. The velocities $v_j(t_l)$ can be treated either as independent values, or to be given functions of time and certain parameters p, e.g. the orbital elements of the spectroscopic binary. Using the standard method of least squares we arrive at the condition

$$0 = \delta S \ , \tag{3}$$

where

$$S = \sum_{l=1}^{k} \int f_l(y) \left| \tilde{I}(y,t_l) - \sum_{j=1}^{n} \tilde{I}_j(y) \exp(iyv_j(t_l;p)) \right|^2 \mathrm{d}y \ , \tag{4}$$

and the functions $f_l(y)$ can be chosen to give the weights of different Fourier modes of the individual spectra. Because S is bilinear in $\tilde{I}_j(y)$ the conditions obtained for them from Eq. (3) are linear equations

$$\sum_{j=1}^{n} \left[\sum_{l=1}^{k} f_l(y) \exp(iy(v_j(t_l;p) - v_m(t_l;p))) \right] \tilde{I}_j(y) = \sum_{l=1}^{k} f_l(y) \exp(-iyv_m(t_l;p)) \tilde{I}(y,t_l) \tag{5}$$

($m = 1, ..., n$), independent for each y. Just this independence of Fourier components, which is the consequence of the form of Eq. (2) local in the variable y (unlike the form of Eq. (1) integral in the variable x), makes the disentangling of the observed spectra easier in Fourier transform then in the wavelength space. Solving this system of equations for each y and substituting $\tilde{I}_j(y)$ into Eq. (4), S can be optimized only with respect to parameters either $v_j(t_l)$ or p.

It is obvious that Eq. (5) is singular for $y = 0$. This corresponds to the fact that the contributions of individual stars to the constant term of $I(x)$ cannot be distinguished. The continua of stars are almost constant and in practice unaffected by the Doppler shift. Consequently, a large error of $\tilde{I}_j(y)$ can be expected for small values of y. It can thus be convenient to cut-off this range of y choosing here $f_l = 0$.

3. Numerical method

The Fourier transform is calculated by FFT (in 256 points in the present version of the code KOREL). The δ-function in Eq. (1) must be discrete with the sampling frequency (to give in Eq. (1) function $\exp(iyv_j(t))$ periodic). Consequently, both $v_j(t)$ and S are step functions (in t and p, resp.). To achieve a good resolution in velocity, it can be thus advantageous to interpolate the input spectra into higher sampling density then is the original one read from the detector. The Fourier transform of these spectra must be stored in the computer memory in the course of the solution. To enable the use of large number of spectra with high resolution, the spectra can be represented by several spectral regions only (containing spectral lines chosen for the solution), each one characterized by the initial wavelength and step in radial velocity per one bin. For each spectral region the function S is given by Eq. (4) and the corresponding sets of Eqs. (5) are independent. The total S summed over spectral regions is calculated only for the purpose of convergence of parameters p.

The spectra are supposed to originate from a multiple stellar system with a hierarchical structure shown in Fig. 1, where the numbers in circles are indexes of the star position, numbers in parenthesis give indexes of the corresponding orbit. The occupation of each position by a visible component is to be given by special key on input. The orbits Nos. 1, 2 and 3 can be suppressed by the choice of the corresponding period equal to 0. The radial velocity

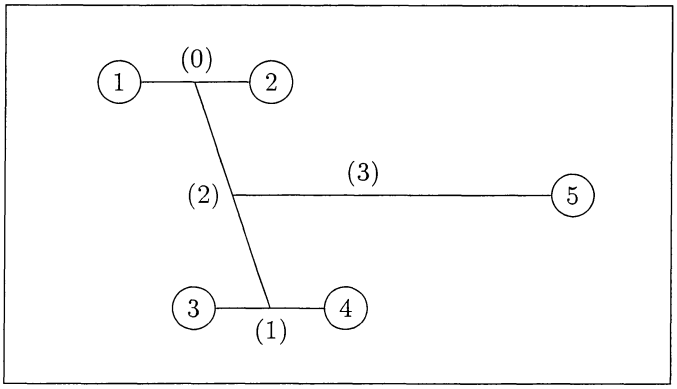

Fig. 1. The structure of the stellar system

of a component is thus given by

$$\mathrm{RV} = \sum_{\mathrm{o}} K(\cos(\omega + v) + e\cos\omega) \,, \qquad (6)$$

where the summation is performed over the orbits influencing the motion of the star (e.g. orbits Nos. 0, 2 and 3 for the star No. 1; note that the γ-velocity of the system does not appear in this formula – it can be specified only after the identification of lines in the component spectra). The true anomaly v is calculated from the solution of the Kepler's equation for the time t corrected by

$$\Delta t = \sum_{\mathrm{o}} \frac{PK}{2\pi c}(1-e^2)^{3/2}\frac{\sin(\omega+v)}{1+e\cos(v)} \,, \qquad (7)$$

for the light-time effect due to the higher orbits in the hierarchical system (e.g. Nos. 2 and 3 for stars Nos. 1 and 2). The spectra and times of exposures are usually transformed into the heliocentric system. If not, the higher orbit can be used to make the corresponding correction. The secondary component of this 'solar' orbit can be used to get the telluric lines away (in an approximation) from the stellar spectra.

The minimization of S with respect to p is performed by the simplex method adapted from Kallrath & Linnel (1987). Several orbital elements (up to 10) can be chosen from all of them (i.e. period $P = p_{1+10\mathrm{o}}$, time of the pericenter passage $t = p_{2+10\mathrm{o}}$, eccentricity $e = p_{3+10\mathrm{o}}$, pericenter longitude $\omega = p_{4+10\mathrm{o}}$, K-velocity $K = p_{5+10\mathrm{o}}$ of the primary, i.e. of the component(s) with lower index, and mass ratio of secondary over primary $q = p_{6+10\mathrm{o}}$ for each orbit o) for convergence in one step, the other being fixed.

4. Results

As an example, the solution of simulated data for a binary star is plotted in Fig. 2. The natural line profiles with central intensities in ratio 4:1 and semi-widths corresponding to radial velocities 20 and 35 km/s are assumed for the primary and secondary, respectively. Profiles calculated (for sampling step 2 km/s) for several phases of circular orbit with $K_1 = 22$ and $K_2 = 44$ km/s (with an artificial noise) are apparently single peaked with a variable asymmetry. However, starting from $K_1 = 10$ km/s (other elements – including the mass ratio – fixed to their correct values), the above described procedure arrives (in a few minutes on PC 286) back to value $K_1 = 22.17$ km/s in 5 steps of the simplex minimization and reproduces well the profiles of both components.

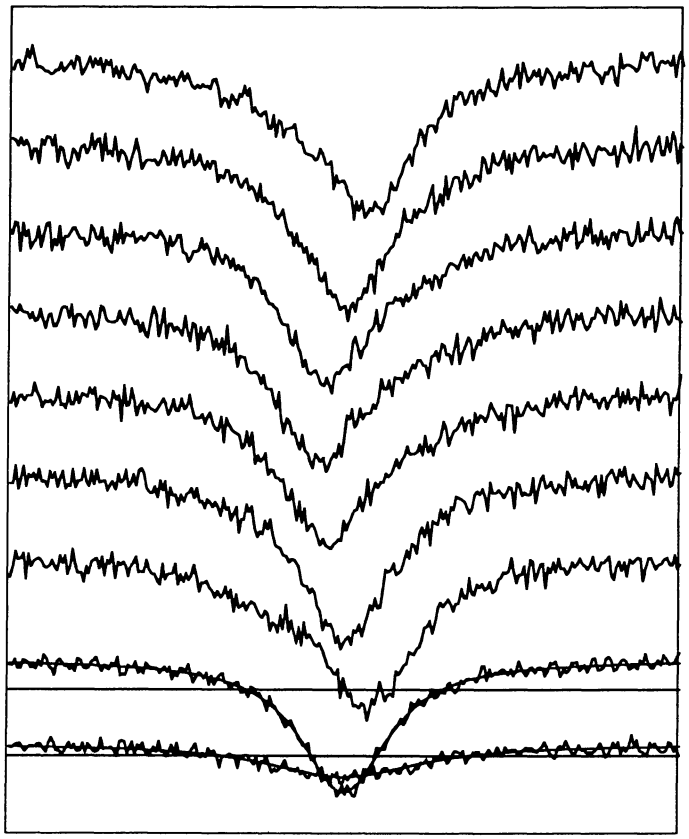

Fig. 2. Simulated composite line profiles (7 upper curves) of a binary and the solution of their disentangling superposed on the input profiles (bottom 2 curves)

Two warnings should be kept in mind of a user of this method. First, the minimization of S with respect to orbital parameters p is performed by a type of local search. Consequently, the solution can be dependent on the initial conditions and need not be necessarily the best solution (it can be a local instead of the global minimum of S). A use of information from other sources (e.g. period obtained from photometry) can significantly help not

only to save the computational time but also to get the proper solution. Second, even the best solution need not be necessarily a good solution! Both the solution of light- and RV- curves by FOTEL and solution of composite spectra and orbital parameters by KOREL fits the input data by the implicitly included model as well as possible, but cannot guarantee that this model is correct. Another model can fit the same data as well or even better. For instance, similar variable asymmetry of a line profile as seen in Fig. 2 can result also from a rotation of a star with inhomogeneities on its surface or of its pulsations. A correspondence of the component spectra obtained by the disentangling with synthetic spectra resulting from model atmospheres can be used as an evidence of plausibility of the multiple-star model for a particular object. Generally, consistency of all the observational results should be verified in a study of the object. This is also why the application of the present method to a real data will be demonstrated in a subsequent paper. Preliminary results from such calculations seems to be promising.

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