Lecture

is available on

http://www.astro.uni-bonn.de/~bertoldi/wiki/RadioInterferometry

Interferometric Calibration & Imaging

Peter Schilke, MPIfR

Editing and calibration

- An Interferometer measures the visibility of antenna pairs (or on baselines)

$$V_{ij}(t) = \iint A_\nu(l,m)\, I_\nu(l,m)\, e^{-2\pi i (u_{ij}(t) l + v_{ij}(t) m)}\, dl\, dm$$

- the term (*ul*+*vm*) gives the geometrical phase difference between the phase center (where the fringes are stopped) and the source
- the phase determines the location of a source
- the amplitude the strength

Editing and calibration

- Editing is basically just throwing out of bad data
- ...so one either has to have good automatic quality checks (phase jumps) and/or good visualization tools
- Calibration is correcting the data for meas-urement errors

Measured in practice

$$\tilde{V}_{ij}(t) = G_{ij}(t)\, V_{ij}(t) + \epsilon_{ij}(t) + \eta_{ij}(t)$$

where

$G_{ij}(t)$ *is a baseline dependent complex gain*

$\epsilon_{ij}(t)$ *is a baseline dependent complex offset*

$\eta_{ij}(t)$ *is a stochastic noise term*

Calibration

- One calibrates by observing a source of known structure – ideally, a point source – once in a while
- “once in a while” depends on the stability of the system
  - instrumental part
  - atmospheric part
- Compromise between calibrating often (good data quality) and calibrating rarely (observing efficiency)

Atmospheric stability

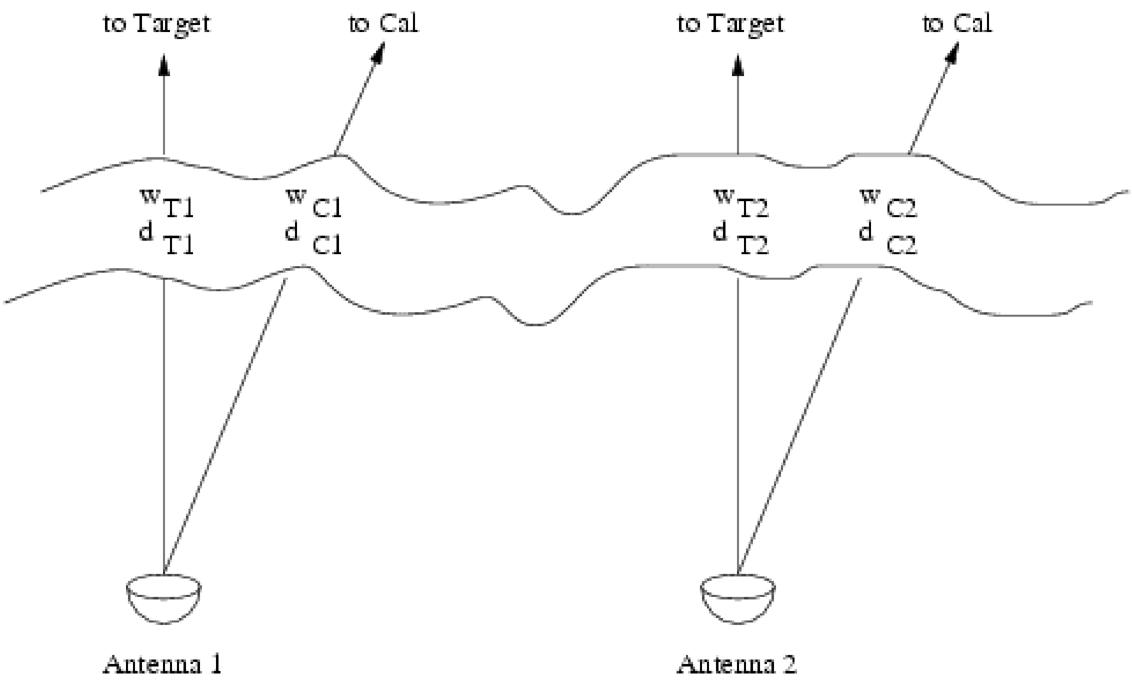
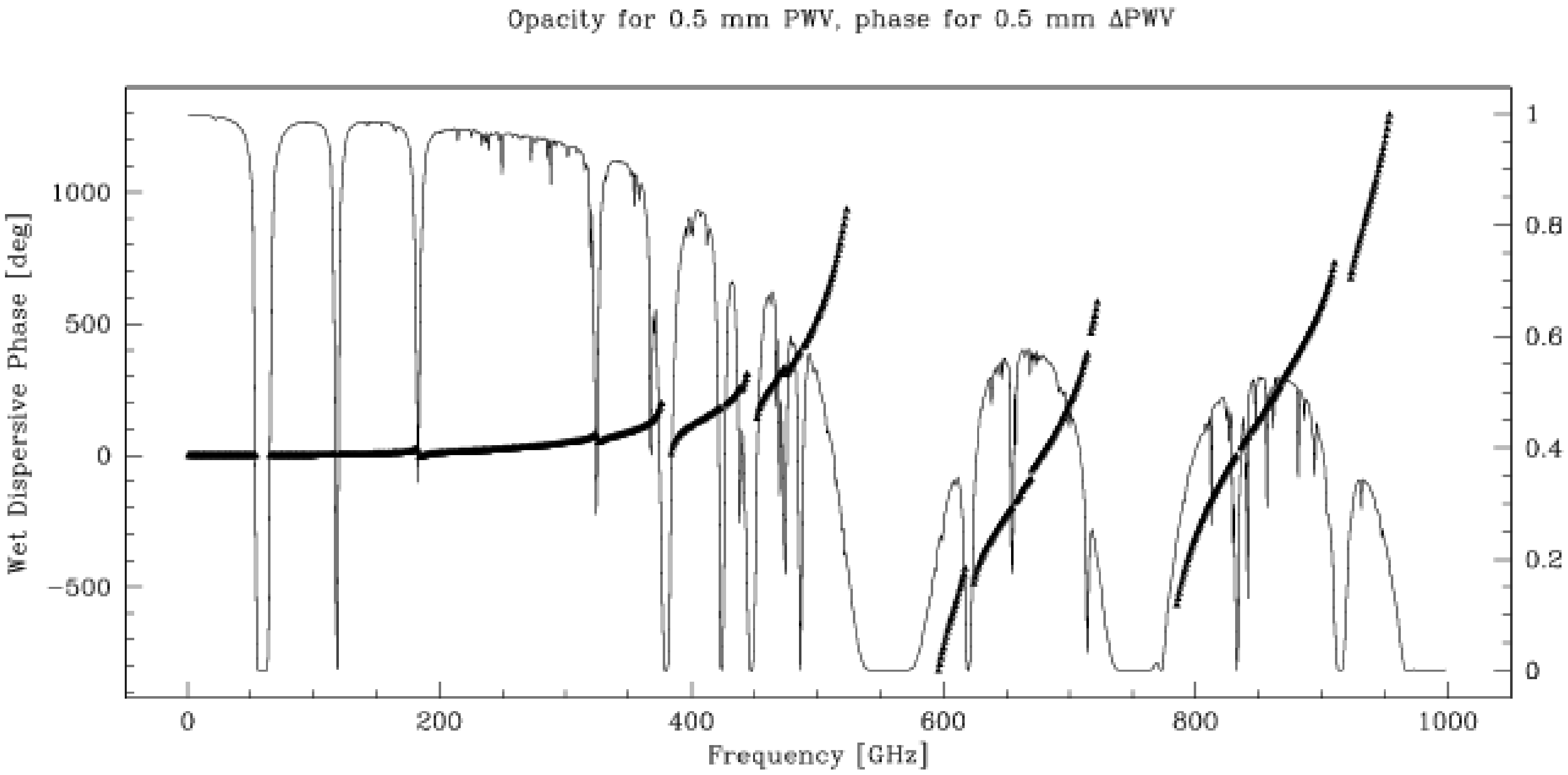


Figure 3: Schematic showing the water vapor columns (*eg*, w_{C1}) and dry delays (*eg*, d_{C1}) toward the calibrator and the target sources above antennas 1 and 2.

Calibration

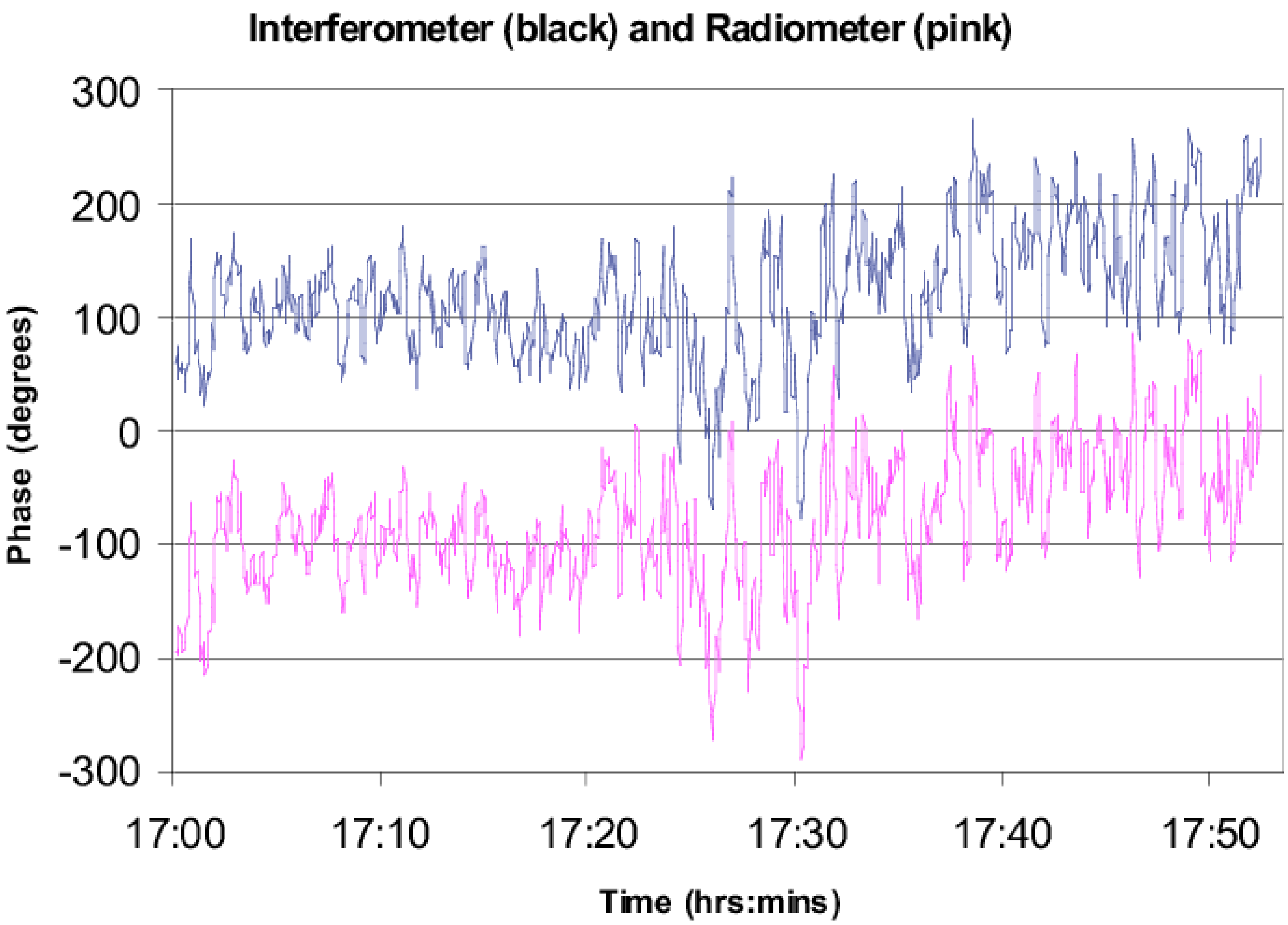
- Other compromises:
  - calibrator source should be strong
    - to get good S/N
  - nearby
    - to minimize dead time for moving telescope
    - to have same atmospheric conditions

Phase dispersion



nondispersive part: $\Delta\phi \propto \nu$

Alternative: radiometric correction



Amplitude and phase calibration

- Baseline based
  - natural – visibilities are measured on baselines
  - tedious – for N antennas there are N (N-1)/2 baselines, e.g. for ALMA with N=50 there are 1225 baselines
    - not possible to do visually
- Antenna based
  - most errors ARE antenna based
  - choice for more than 5-6 antennas out of ne-cessity

Calibration – baseline based

- For a point source with complex visibility *S* (i.e. Amplitude is *S* Jy and phase is 0), one gets

$$G_{ij}(t) = \frac{\tilde{V}_{ij}(t)}{S}$$

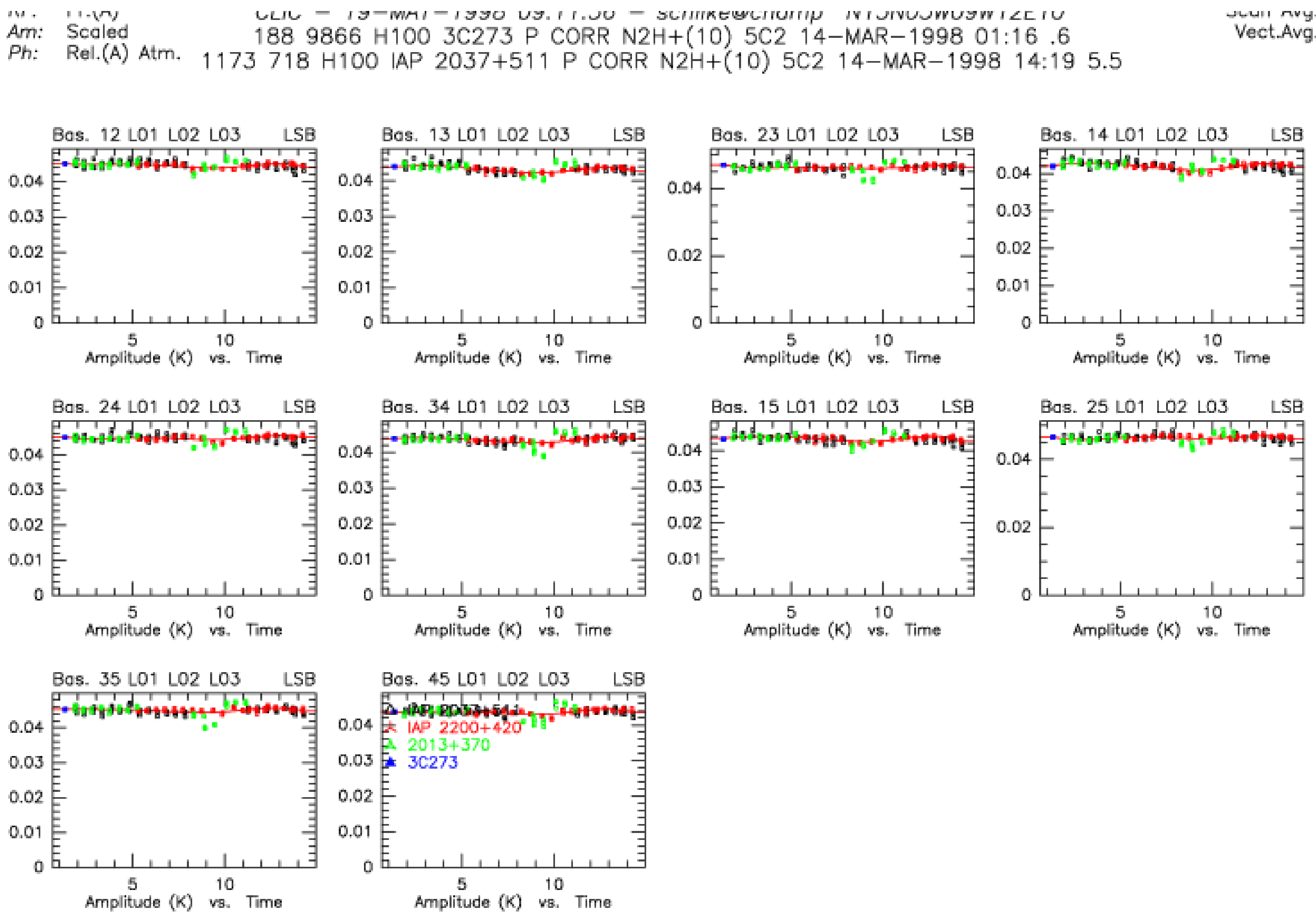
if

$\epsilon_{ij}(t)$ and $\eta_{ij}(t)$ *can be neglected*

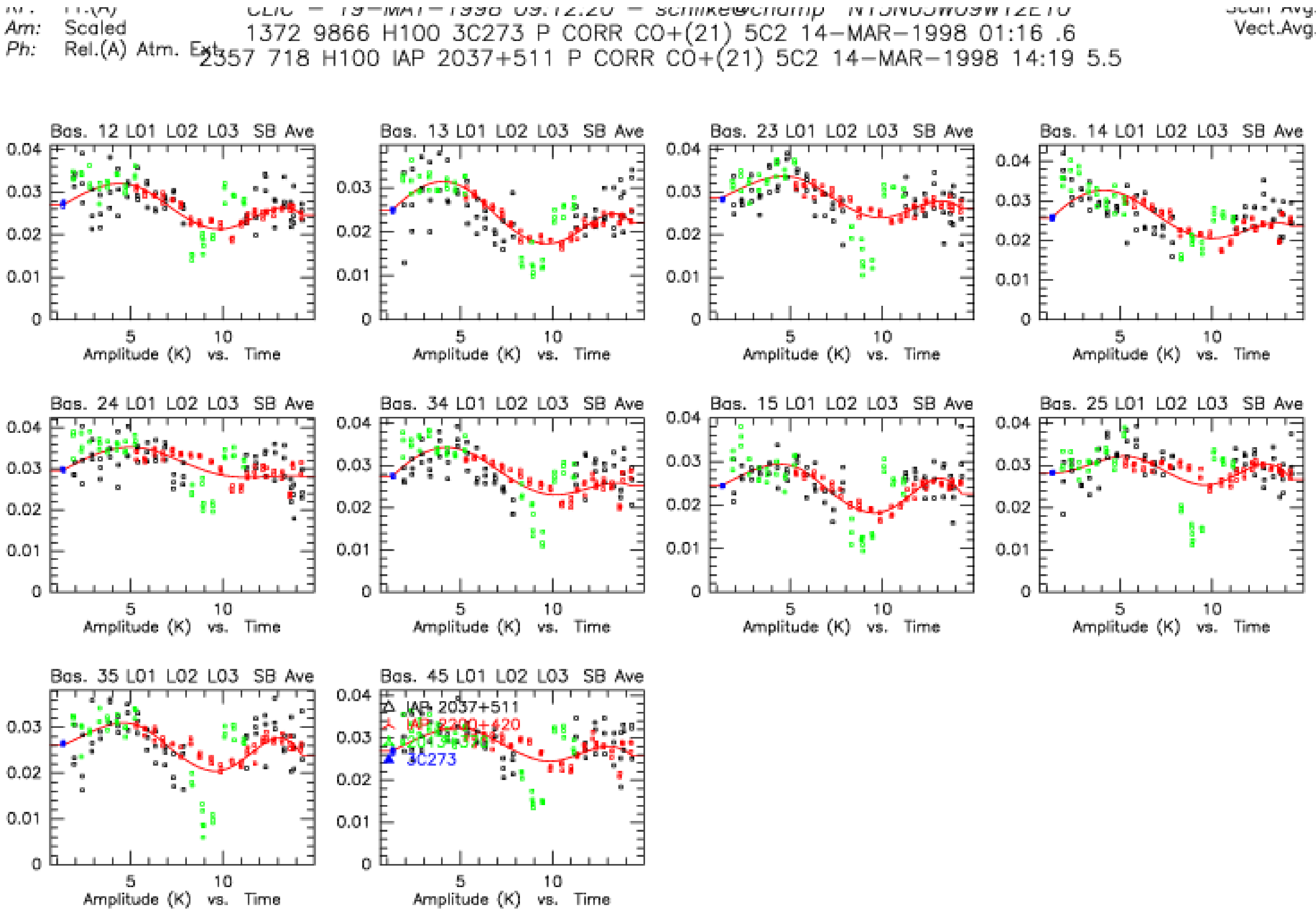
Phase and Amplitude calibration

- This $G_{ij}(t)$ is measured from time to time – for each baseline – interpolated in time and applied to the source
- sometimes one uses more than one calib-rator (left and right of the source)
- For high frequencies, there often are no strong calibrators closeby – phase transfer can then be employed

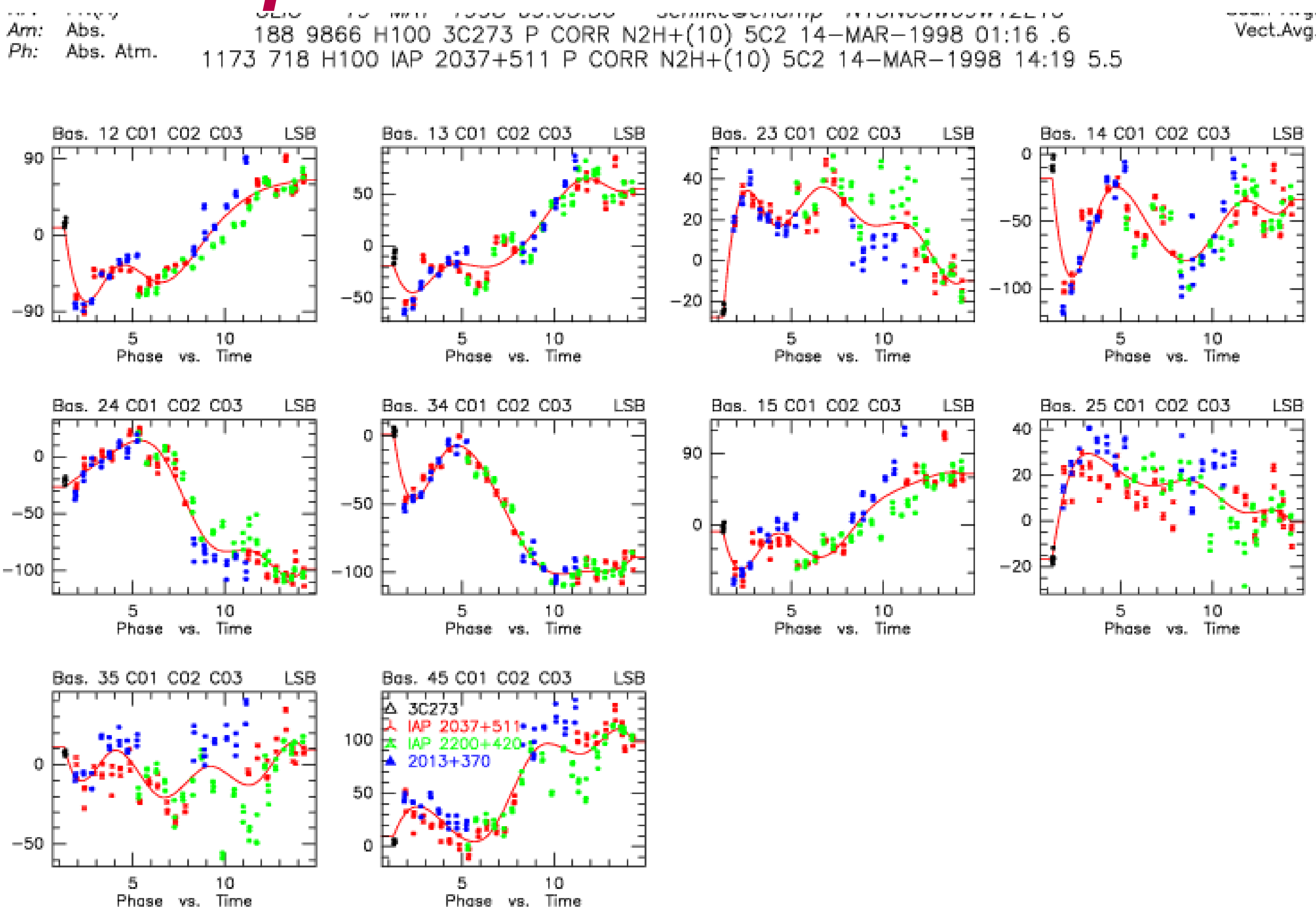
Example: PdB amplitude calibration - 3mm



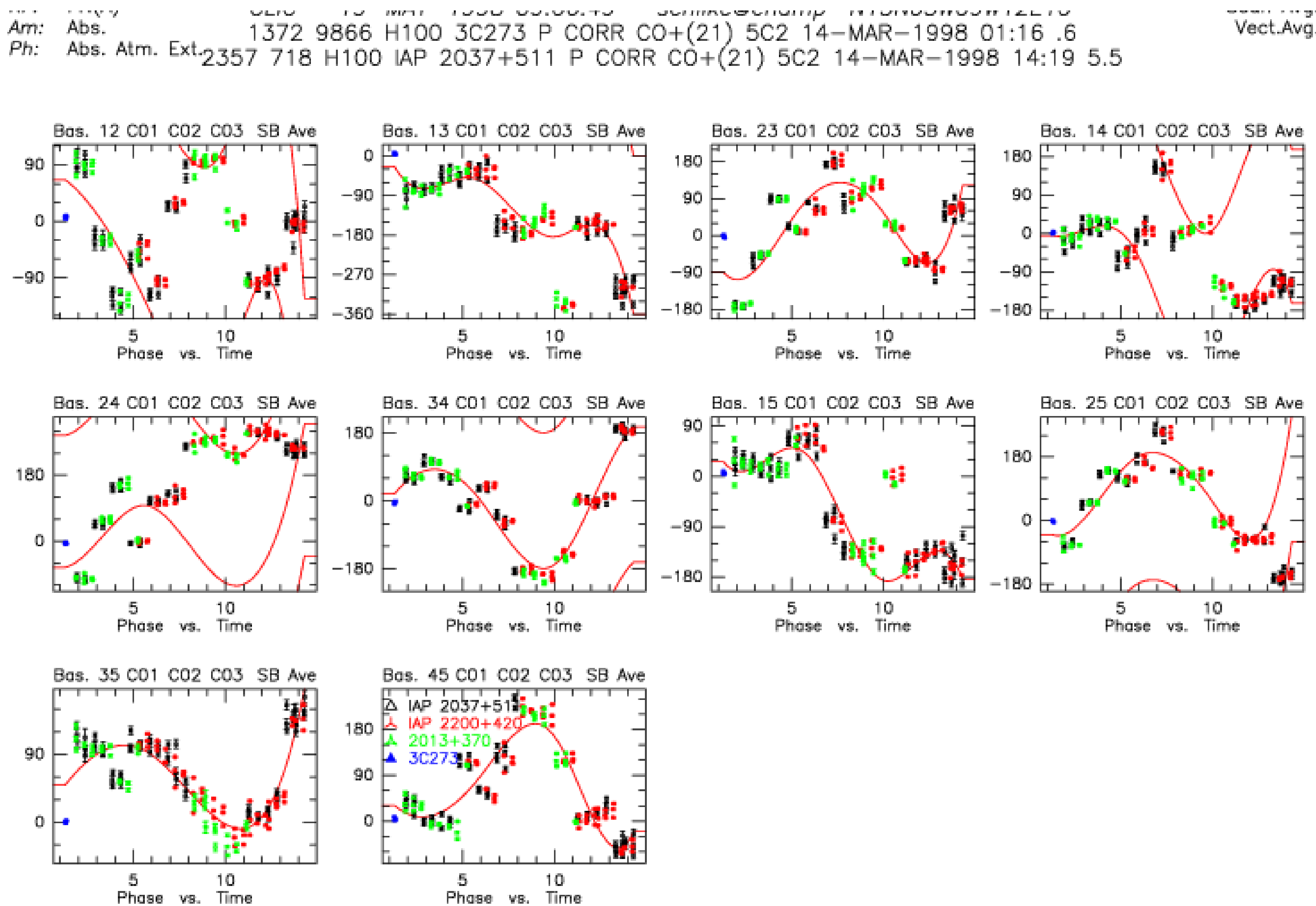
PdB amplitude calibration - 1mm



PdB phase calibration - 3mm



PdB phase calibration - 1mm



Antenna based calibration

$$G_{ij}(t) = g_i(t)\, g_j^*(t)\, g_{ij}(t)$$

- g_i and g_j are the antenna based gains, g_{ij} is the baseline dependent residual (closure error), which usually is in the 1% range
- this leads to the amplitude and phase equations

$$A_{ij}(t) = a_i(t)\, a_j^*(t)\, a_{ij}(t)$$
$$\Phi_{ij}(t) = \phi_i(t) - \phi_j(t) + \phi_{ij}(t)$$

Antenna based calibrations

- real and measued visibilities are then

$$V_{ij}(t) = A_{ij} e^{i\phi_{ij}}$$

$$\tilde{V}_{ij}(t) = \tilde{A}_{ij} e^{i\tilde{\phi}_{ij}}$$

for a point like calibrator with flux density S

$$A_{ij} = S \text{ and } \phi_{ij} = 0$$

and

$$\tilde{A}_{ij} = a_i a_j A_{ij} S$$

$$\tilde{\phi}_{ij} = \phi_i - \phi_j + \phi_{ij}$$

- which can be solved when the closure error is small

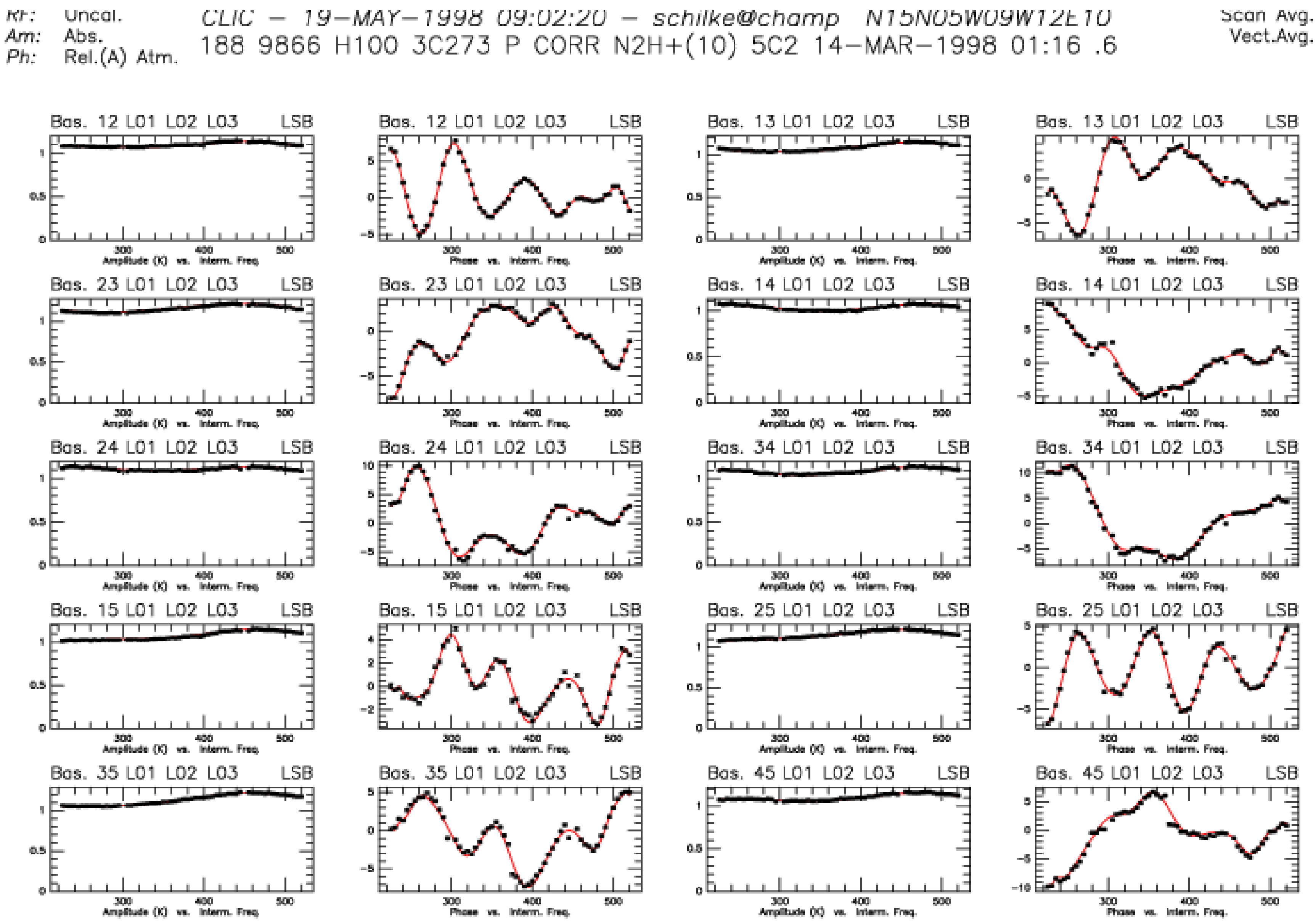
Phase calibration

- For the phase, the equation depends on the differences of phases
- one antenna is used as reference antenna with φ_i=0

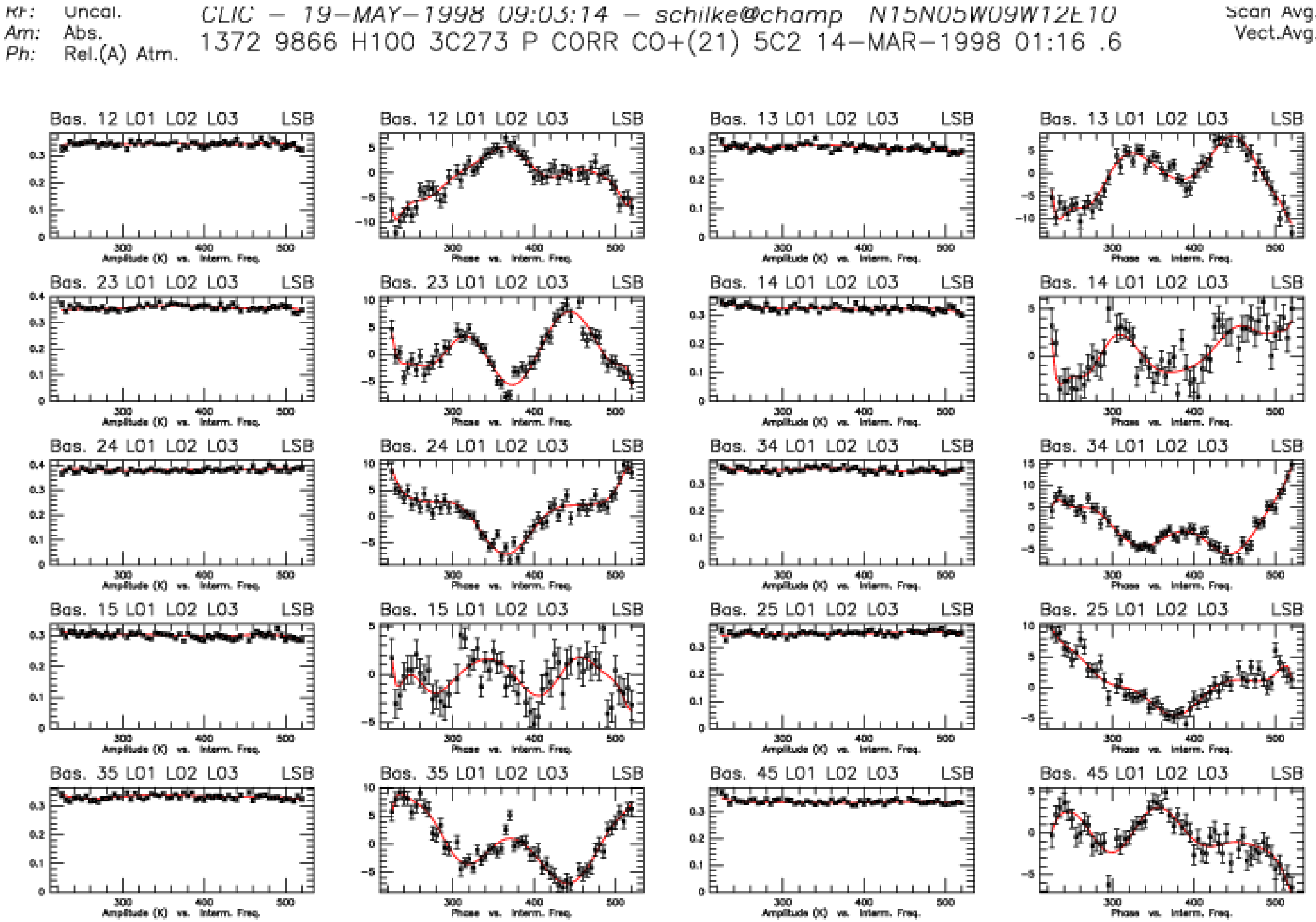
Bandpass and Flux calibration

- Bandpass
  - For spectral lines observations, one also has to calibrate the bandpass
  - use line-free, strong source and do long integra-tion
- flux calibration
  - use source of known flux
  - tricky: planets are too big, they're resolved out
  - quasars are time variable

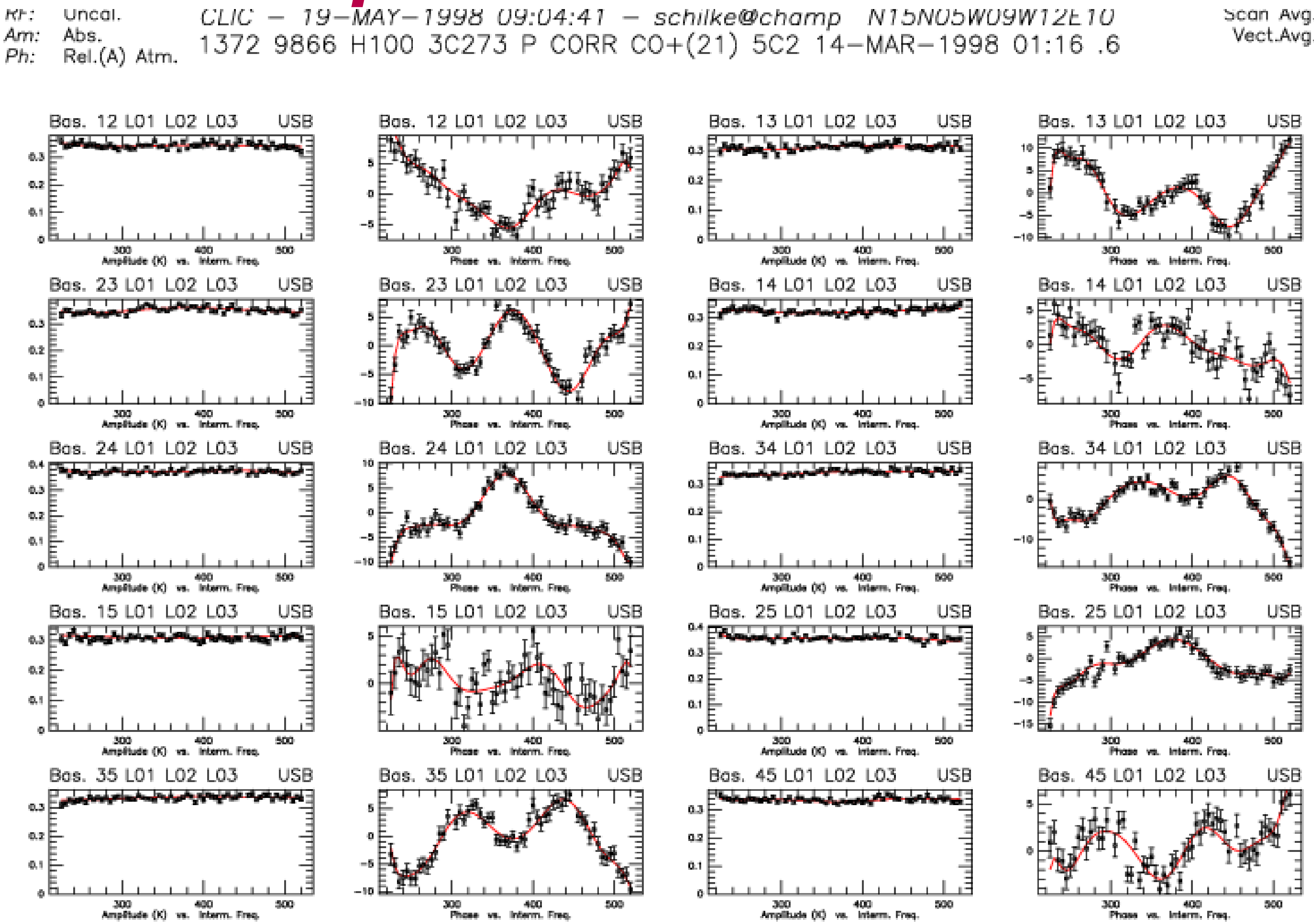
Example bandpass PdB - 3mm



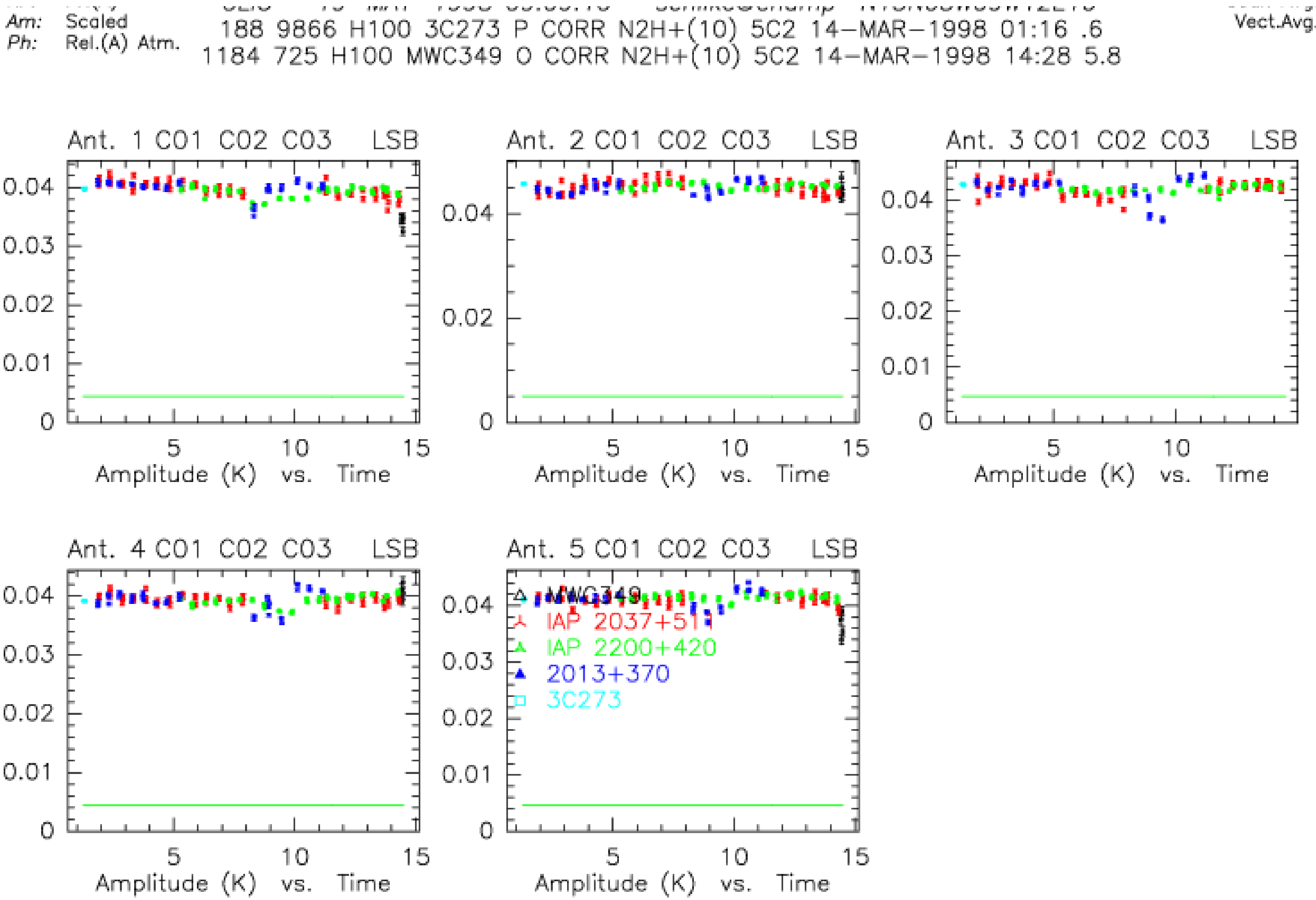
Bandpass PdB 1mm - LSB



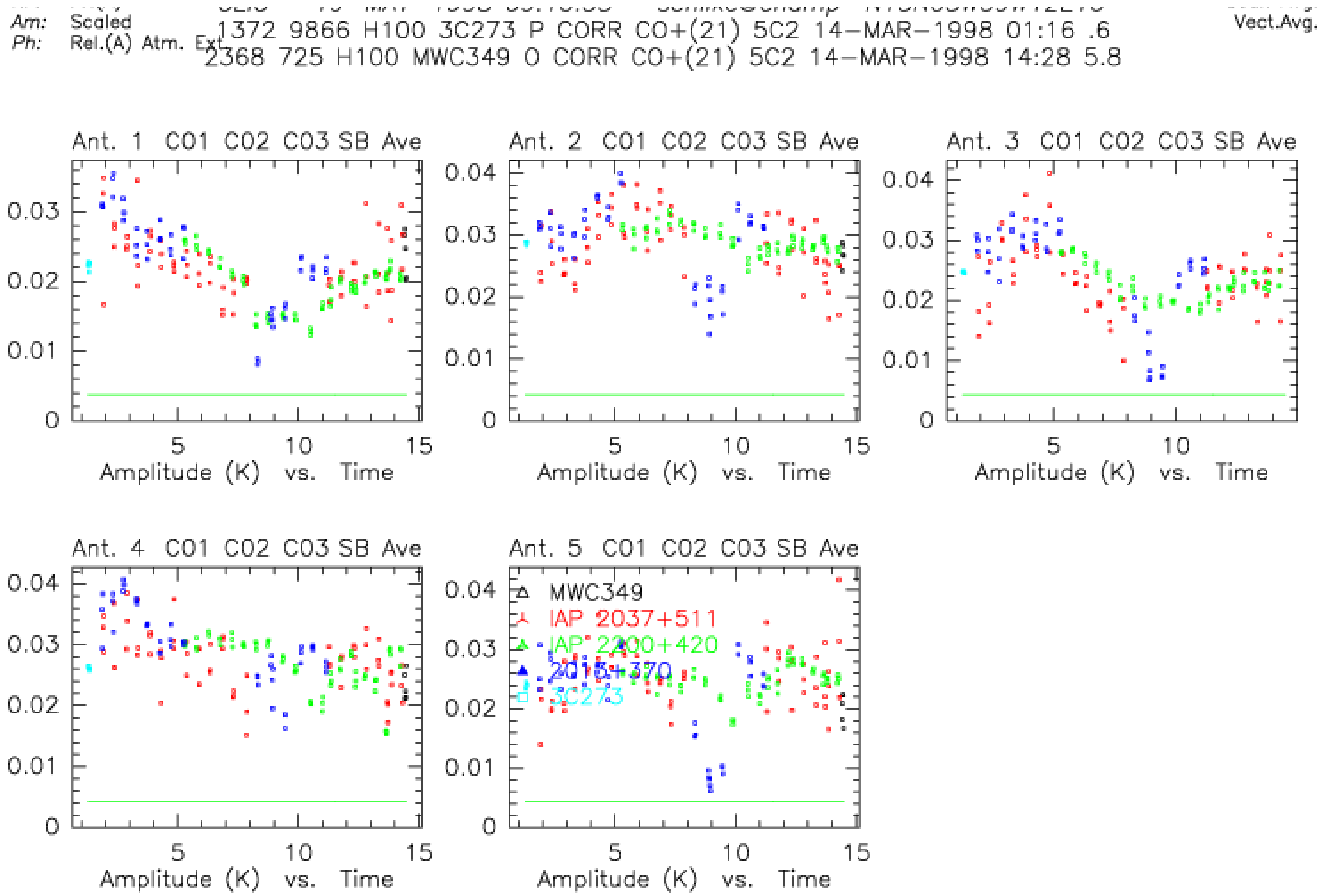
Bandpass PdB 1mm - USB



Flux calibration PdB 3mm



Flux calibration PdB 1mm



Interferometric Calibration & Imaging

Peter Schilke, MPIfR

Lecture

is available on

http://www.astro.uni-bonn.de/~bertoldi/wiki/RadioInterferometry

Imaging

$$A_\nu(l,m)\,I_\nu(l,m)=\iint V_\nu(u,v)\,e^{-2\pi i(ul+vm)}\,du\,dv$$

in reality, with incomplete sampling

$$I_\nu^D(l,m)=\iint V'_\nu(u,v)\,S(u,v)\,e^{-2\pi i(ul+vm)}\,du\,dv$$

direct Fourier transform of all M points of the (u,v) *plane*

$$I_\nu^D(l,m)=\frac{1}{M}\sum_{k=1}^{M} V'_\nu(u,v)\,e^{-2\pi i(u_k l+v_k m)}$$

Imaging

- and this on an *NxN* raster (in the sky plane)
- this requires $4MN^2$ multiplications or, since M is of order N^2, $O(N^4)$ multiplications
- since N is of order 1000 or so, and one has to multiply with the number of spectral channels, this is a lot of calculations
- Alternative: interpolating on rectangular grid (gridding) and using Fast Fourier Trans-form, which is $O(N^2 log_2 N)$

Sampling

Sampling function

$$S(u,v)=\sum_{k=1}^{M}\delta(u-u_k,v-v_k)$$

sampled visibility function

$$V^S(u,v)=\sum_{k=1}^{M}\delta(u-u_k,v-v_k)V'(u_k,v_k)$$

Fourier transform of sampling function is the dirty beam
folding of true image with dirty beam gives dirty image

Weighted sampling

Weighted sampling function

$$W(u,v)=\sum_{k=1}^{M} R_k T_k D_k \delta(u-u_k, v-v_k)$$

weighted sampled visibility function

$$V^W(u,v)=\sum_{k=1}^{M} R_k T_k D_k \delta(u-u_k, v-v_k) V'(u_k, v_k)$$

Weighting terms

- R_k: weighting for quality of data
  - system temperature
  - integration time
  - bandwidth
- T_k: tapering function
  - downweighting of outer points
    - cleaner, but larger beam
- D_k: density weighting
  - takes into account clumping of data
    - $D_k=1$: natural weighting: better sensitivity
    - $D_k=1/N_s(k)$: uniform weighting: $N_s(k)$ is number of data points in region of width *s:* higher resolution

Gridding

- Folding with function C

$$V^R(u,v)=\sum_{k=1}^{M} C(u_c-u_k, v_c-v_k)\,V^W(u_k,v_k)$$

where C is a function which is identical to *zero outside a small region*

$$V^R(u,v)=R(C*W)=R(C*(WV'))$$

where R is a resampling function

$$R(u,v)=\sum_{j=-\infty}^{\infty}\sum_{k=-\infty}^{\infty}\delta(j-u/\Delta u, k-v/\Delta v)$$

where Δu and Δv define the cell size

Resampling

using the convolution theorem

$$\tilde{I}^D = \overline{V^R} = \overline{R} * (\overline{C}\,\overline{V^W}) = \overline{R} * [\overline{C} * (\overline{W}\,\overline{V}\,')]$$

R is its own fourier transform

$$\overline{R}(l,m) = \Delta u\,\Delta v \sum_{j=-\infty}^{\infty} \sum_{k=-\infty}^{\infty} \delta(j - l/\Delta u, k - m/\Delta v)$$

Resampling

- Resampling makes a periodic function out of I_D, with period *1/Δu* in *l* and *1/Δv* in *m*
- introduces *aliasing*: sources outside the field of view are folded into the field of view (through sidelobes of the dirty beam)
- One has to do the *gridding correction*:

$$\tilde{I}_c^D(l,m)=\frac{\tilde{I}^D(l,m)}{\overline{C}(l,m)}$$ *for the dirty image*

$$\tilde{B}_c^D(l,m)=\frac{\tilde{B}^D(l,m)}{\overline{C}(l,m)}$$ *for the dirty beam*

Gridding functions

- Hat box (square)
  - advantage: fast
  - disadvantage: bad sidelobes, lots of aliasing
- exponential, Gaussian:
  - suppression of aliasing
- Sinc (fourier transform of hat box, i.e. hat box in fourier plane)
- exponential times sinc

Gridding

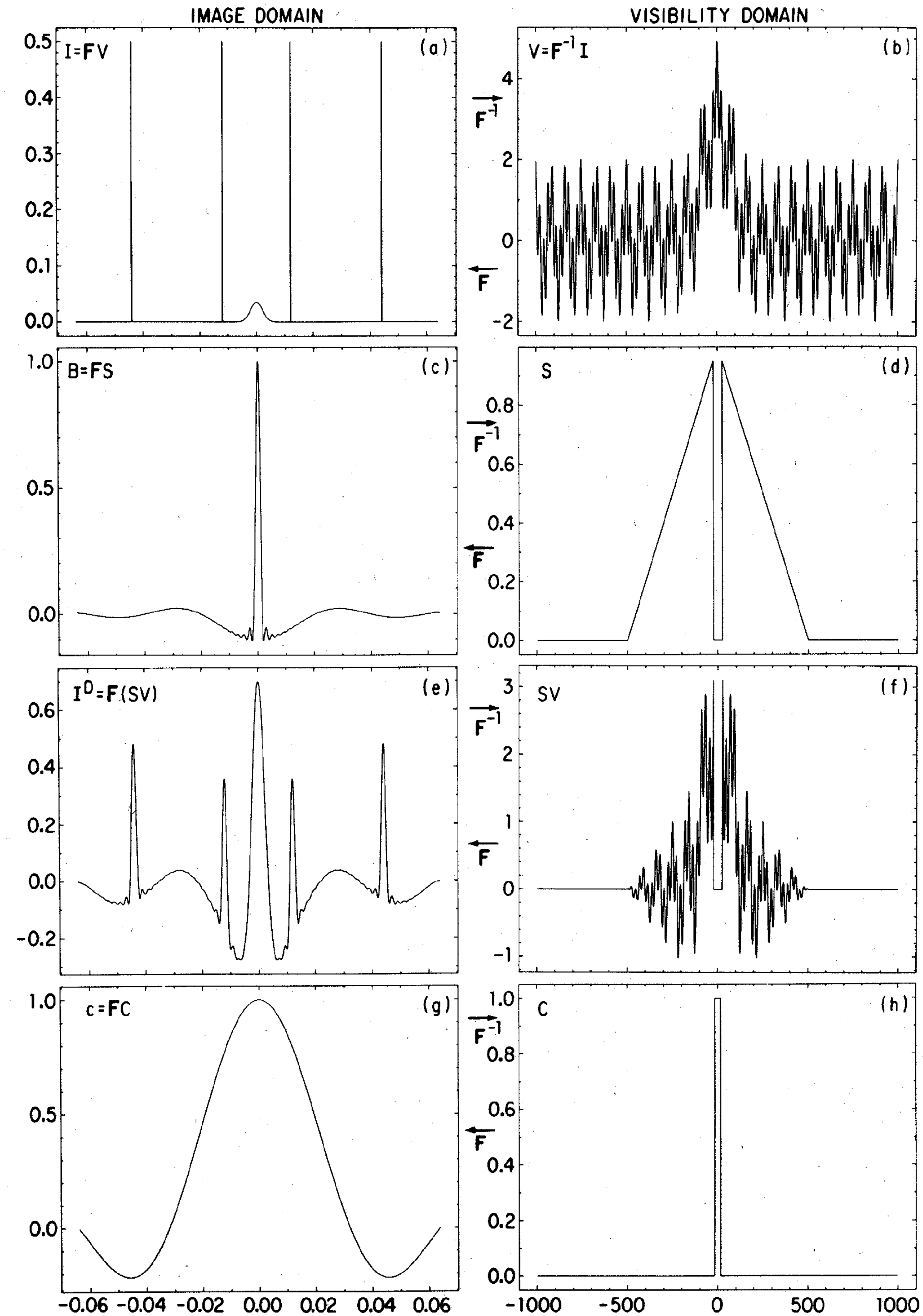


IMAGE DOMAIN
VISIBILITY DOMAIN
I=FV
(a)
V=F^-1 I
(b)
B=FS
(c)
S
(d)
I^D=F(SV)
(e)
SV
(f)
c=FC
(g)
C
(h)
F^-1
F
-0.06 -0.04 -0.02 0.00 0.02 0.04 0.06
-1000 -500 0 500 1000

Gridding

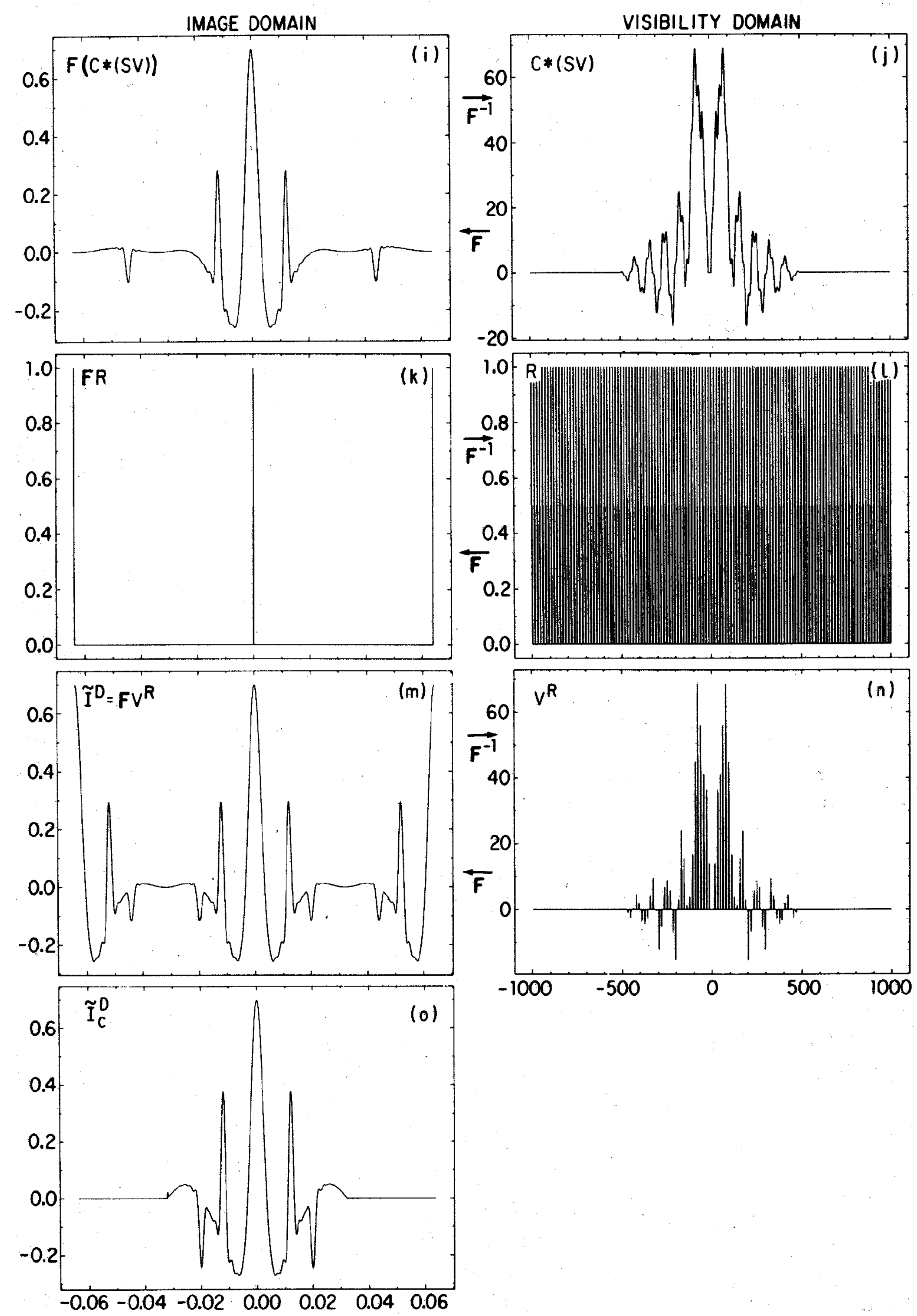


IMAGE DOMAIN
VISIBILITY DOMAIN
F(C*(SV))
(i)
C*(SV)
(j)
FR
(k)
R
(l)
$\tilde{I}^D = FV^R$
(m)
V^R
(n)
$\tilde{I}^D_C$
(o)
F
F^{-1}
0.6
0.4
0.2
0.0
-0.2
60
40
20
0
-20
1.0
0.8
-1000
-500
500
1000
-0.06
-0.04
-0.02
0.00
0.02
0.04
0.06

Visibility sampling

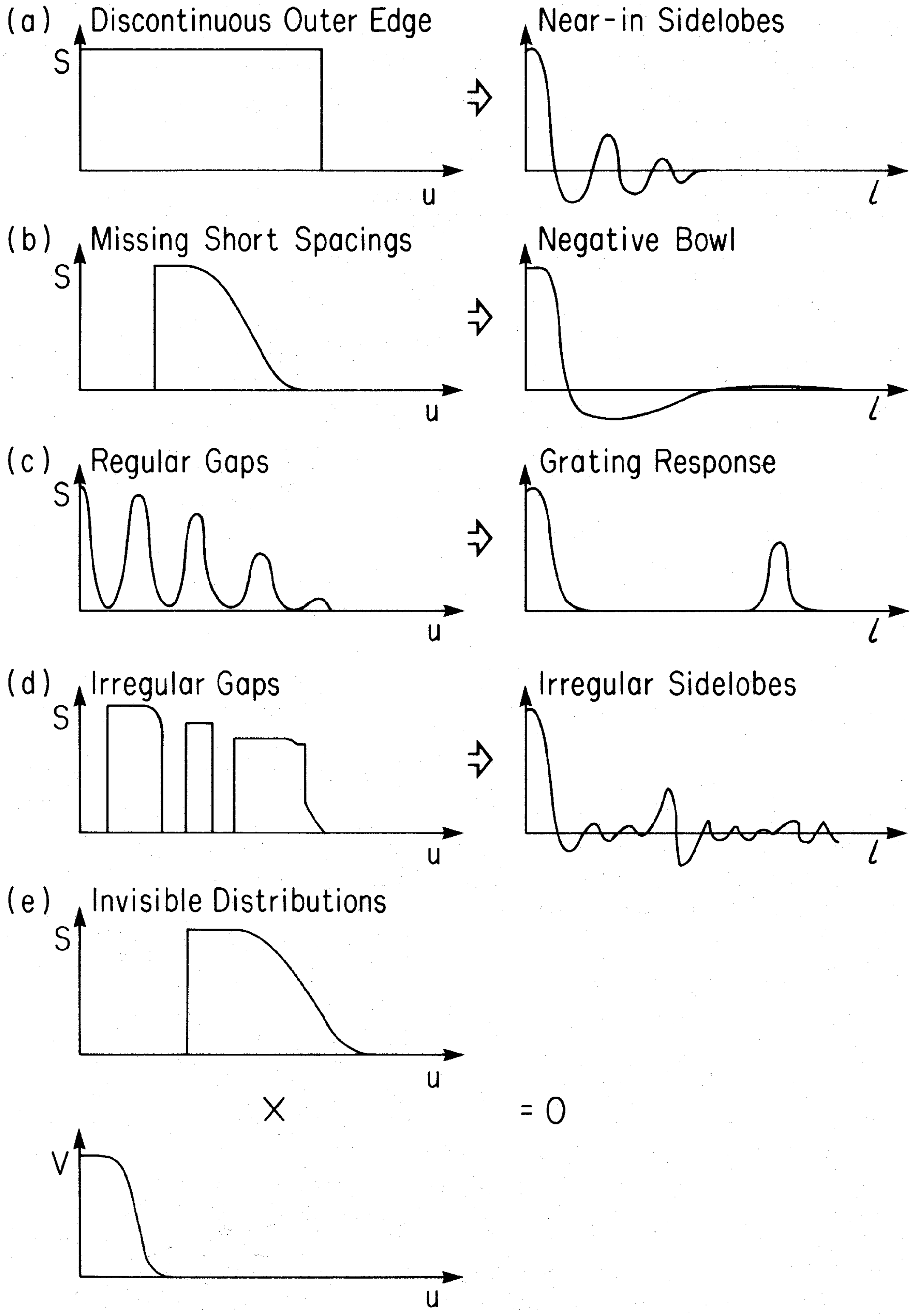


Image restauration

- Goal: “true” image of the sky
- Problems
  - convolution with dirty beam
  - missing spatial frequencies
- Restoration algorithms
  - CLEAN type
  - MEM type

CLEAN

- find amplitude and coordinates of maximum in dirty image (can be restricted to certain areas)
- subtract dirty beam from position of maximum, multiplied with amplitude times loop gain (< 1)
- repeat until residuum is below a certain amplitude limit or maximum number of components has been reached
- convolve all found components with clean beam (elliptical Gaussian fitted to center of dirty beam) and add them up
- add residuum of dirty map

MEM

- Uses model, convolves with dirty beam and minimizes

$$\chi^2 = \frac{\sum_r \left| V(u_r, v_r) - \hat{V}(u_r, v_r) \right|^2}{\sigma^2_{V(u_r, v_r)}}$$

while at the same time minimizing the Entropy

$$H = -\sum_k I_k \ln I_k$$

- to select the “simplest” model compatible with the data

Comparison

- CLEAN is well suited for small arrays, but deals badly with extended emission
  - new developments: multiscale clean
- MEM is computing intensive, but can use models (e.g. from single dish maps) and does extended emission well
- hybrids do exist

UV fitting

- To avoid problems with gridding etc. one can also fit source models directly in the (u,v) plane
- works well, but only for simple geometries (Gaussians, disks, toroids)

Sensitivity

- Sensitivity of single dish telescope

$$\Delta T_A = \frac{f\, T_{sys}}{\sqrt{t\, \Delta \nu}}$$

- where f depends on the measurement method (e.g. $f = \sqrt{2}$ for position switching), also absorbs efficiency losses from clipping and A2D conversion in correlators

Sensitivity

- or, in flux density units

$$\Delta S_\nu = \frac{f\, 2\, k\, T_{sys}}{A_e \sqrt{t\, \Delta\nu}}$$

- for interferometers there are *N* telescopes and *N(N-1)/2* baselines and correlations and each correlation increases the noise by a factor √2

$$\Delta S_\nu = \frac{f\, 2\, k\, T_{sys}}{A_e \sqrt{N(N-1)\, t\, \Delta\nu}}$$

Sensitivity

- Inserting numbers, one finds

$$\Delta S_\nu = 1.44 \frac{f\, T_{sys}}{A_e \sqrt{N(N-1)\, t\, \Delta\nu}}$$

$$\Delta T_b = 19.2 \frac{f\, \lambda^2\, T_{sys}}{A_e \theta^2 \sqrt{N(N-1)\, t\, \Delta\nu}}$$

with λ in *mm*, θ in *arcsec* and $\Delta\nu$ in *kHz*

Units for ΔS_ν *are Jy/beam* and *for* ΔT_b *Kelvin*

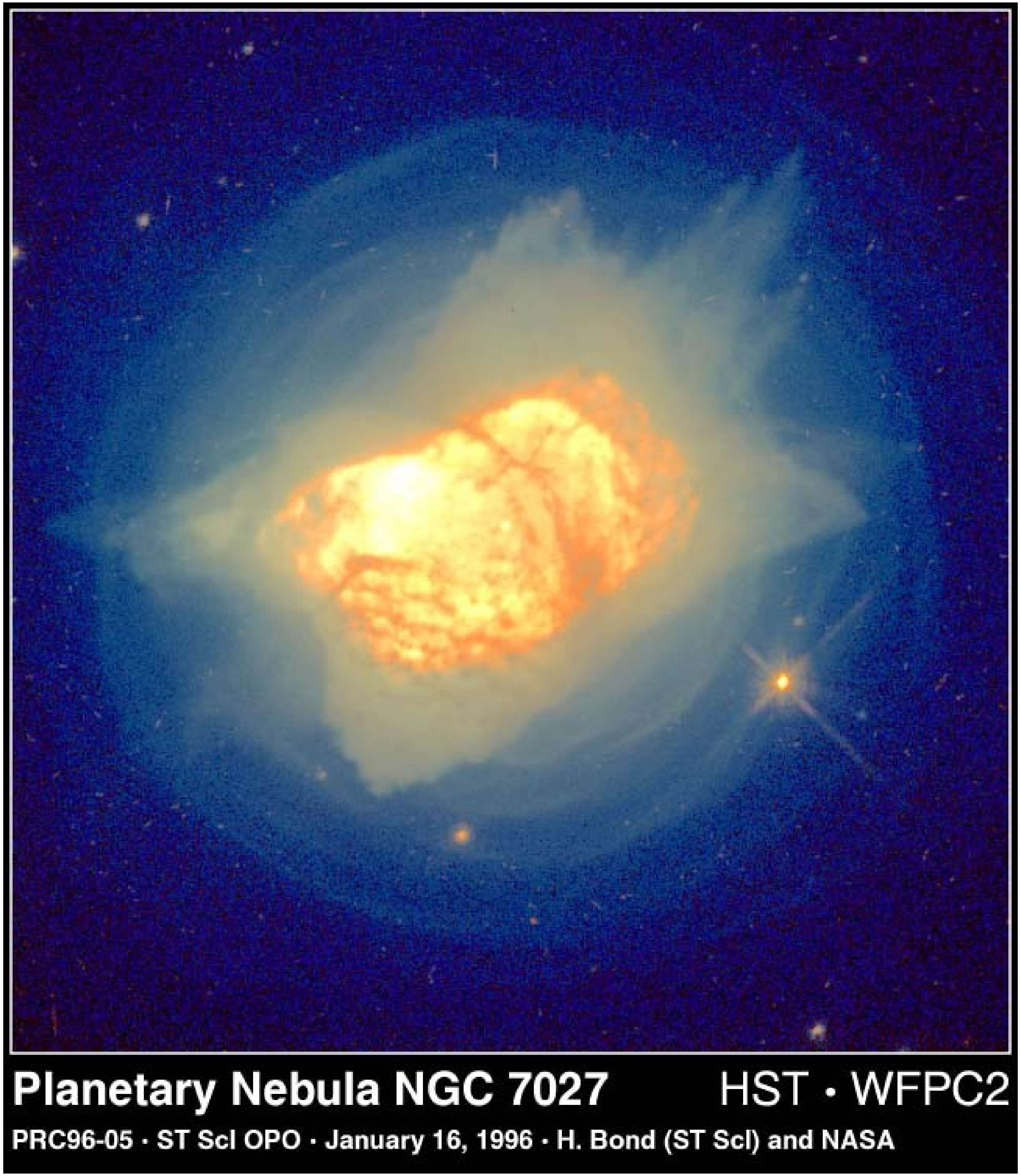
Sensitivities

- Sensitivities in flux depend only on antenna area, integration time and bandwidth
- Sensitivities in temperature depend on beam size, for very small beam size one has a very low brightness temperature sensitivity

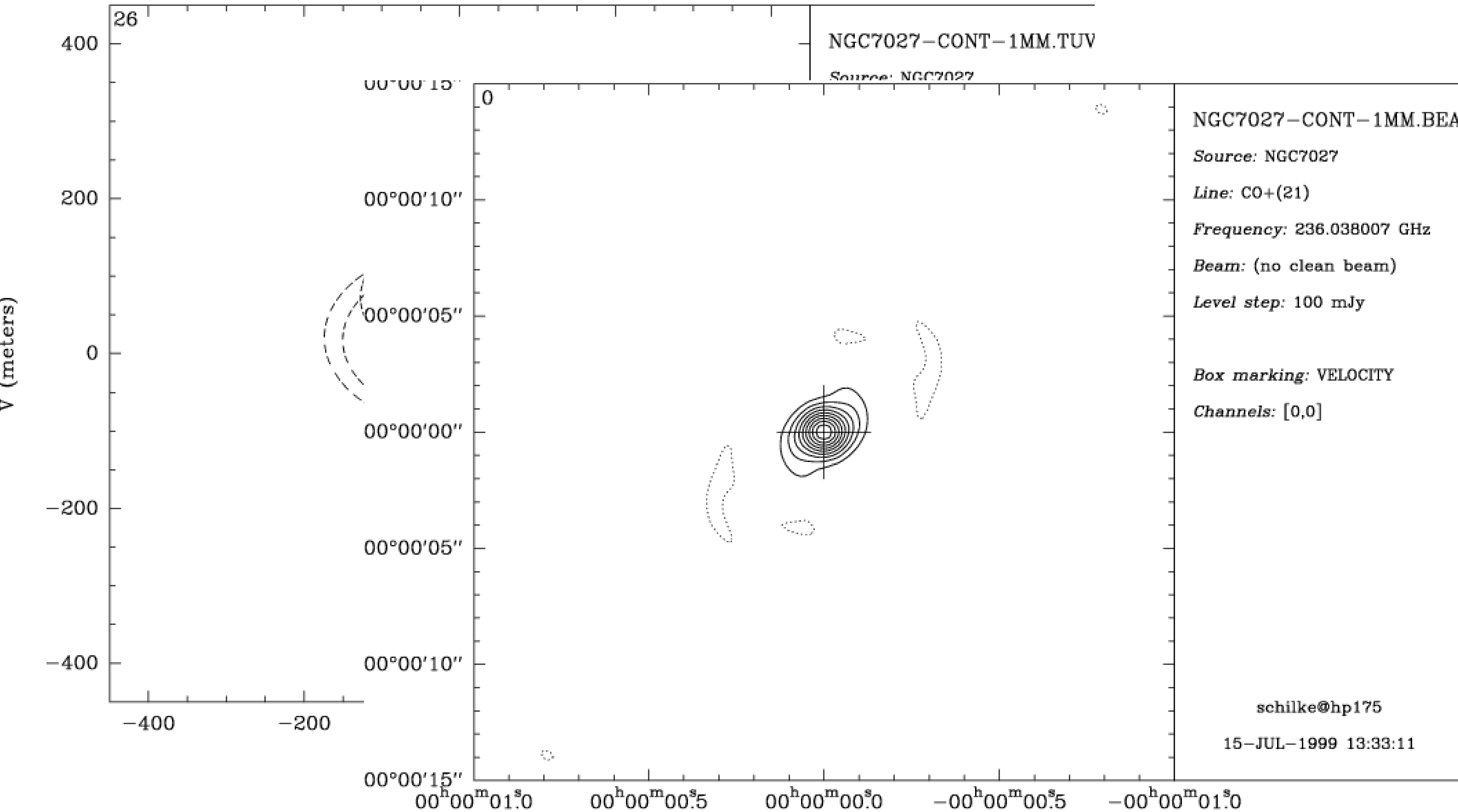
Examples

- Two examples from Plateau de Bure
  - NGC7027
    - planetary nebula at high declination
    - good (u,v) coverage
  - G10.47
    - high-mass star forming region at low declination (-20°)
    - bad (u,v) coverage

NGC7027 optical

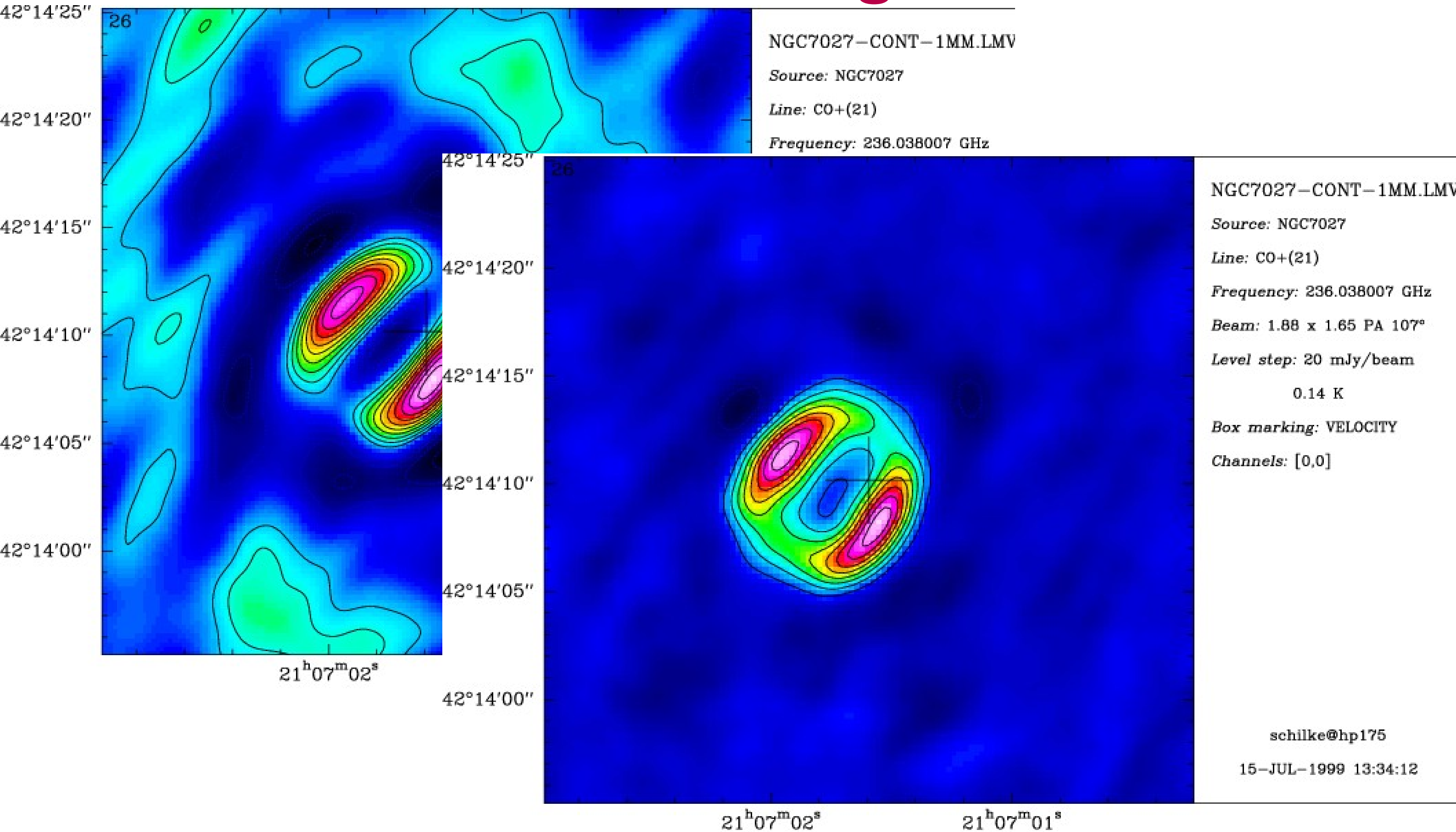


Good beam

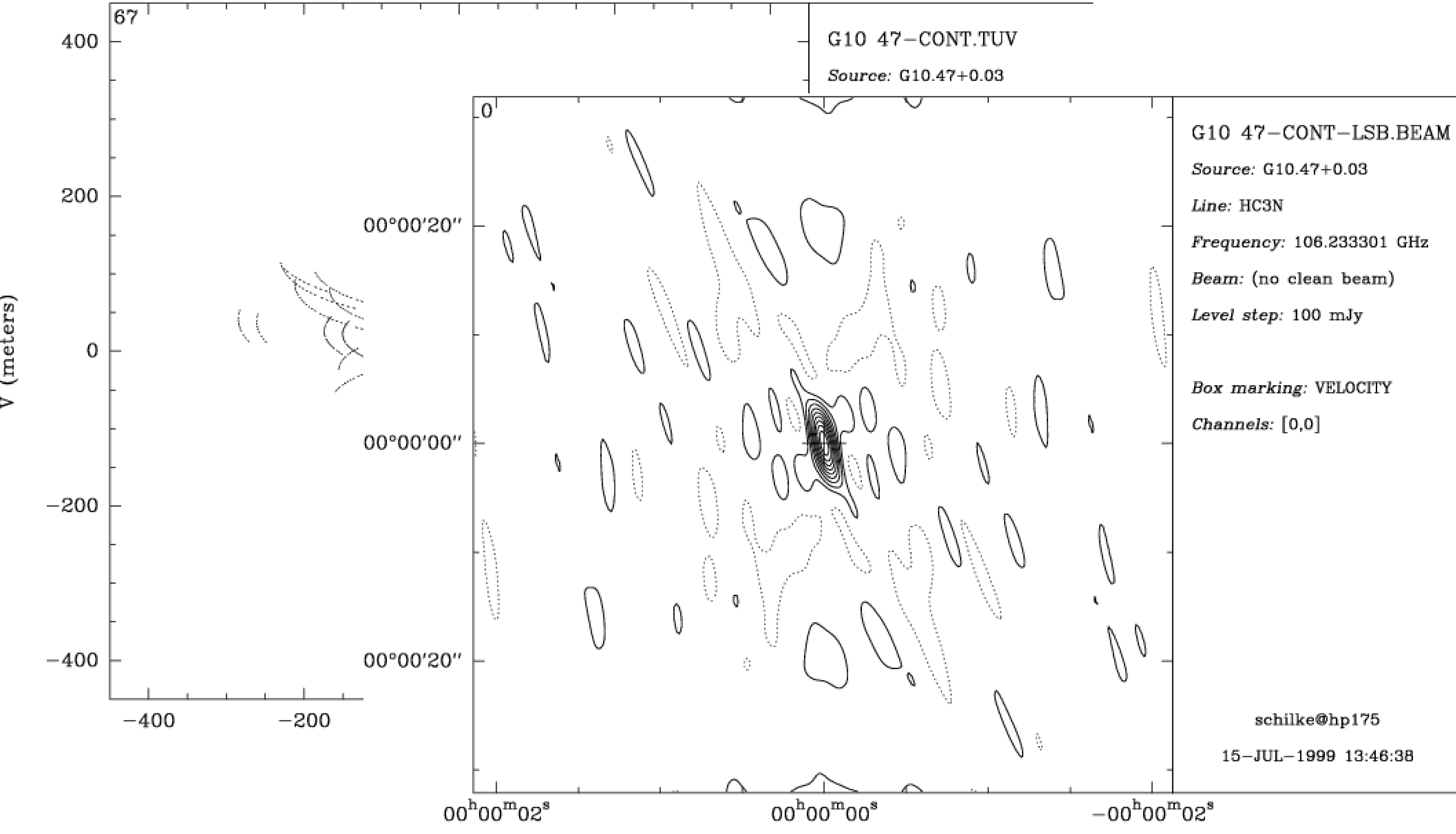


26
400
200
0
−200
−400
−400
−200
V (meters)
NGC7027−CONT−1MM.TUV
00°00'15"
0
00°00'10"
00°00'05"
00°00'00"
00°00'05"
00°00'10"
00°00'15"
00h00m01.0s
00h00m00.5s
00h00m00.0s
−00h00m00.5s
−00h00m01.0s
NGC7027−CONT−1MM.BEA
Source: NGC7027
Line: CO+(21)
Frequency: 236.038007 GHz
Beam: (no clean beam)
Level step: 100 mJy
Box marking: VELOCITY
Channels: [0,0]
schilke@hp175
15−JUL−1999 13:33:11

Good image

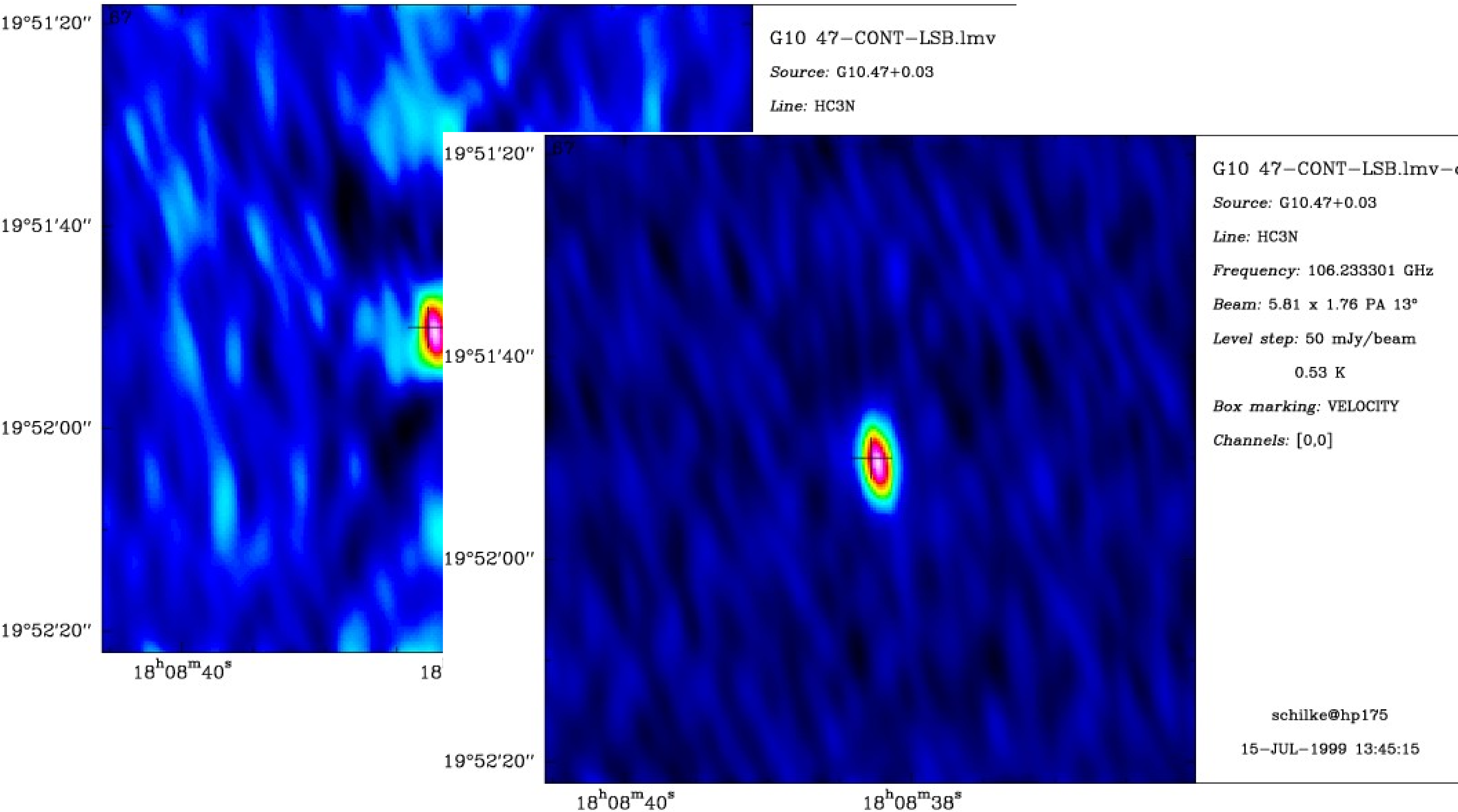

42°14'25"
42°14'20"
42°14'15"
42°14'10"
42°14'05"
42°14'00"
26
21h07m02s
NGC7027–CONT–1MM.LMV
Source: NGC7027
Line: CO+(21)
Frequency: 236.038007 GHz
42°14'25"
42°14'20"
42°14'15"
42°14'10"
42°14'05"
42°14'00"
26
NGC7027–CONT–1MM.LMV
Source: NGC7027
Line: CO+(21)
Frequency: 236.038007 GHz
Beam: 1.88 x 1.65 PA 107°
Level step: 20 mJy/beam
0.14 K
Box marking: VELOCITY
Channels: [0,0]
schilke@hp175
15–JUL–1999 13:34:12
21h07m02s
21h07m01s

Bad beam



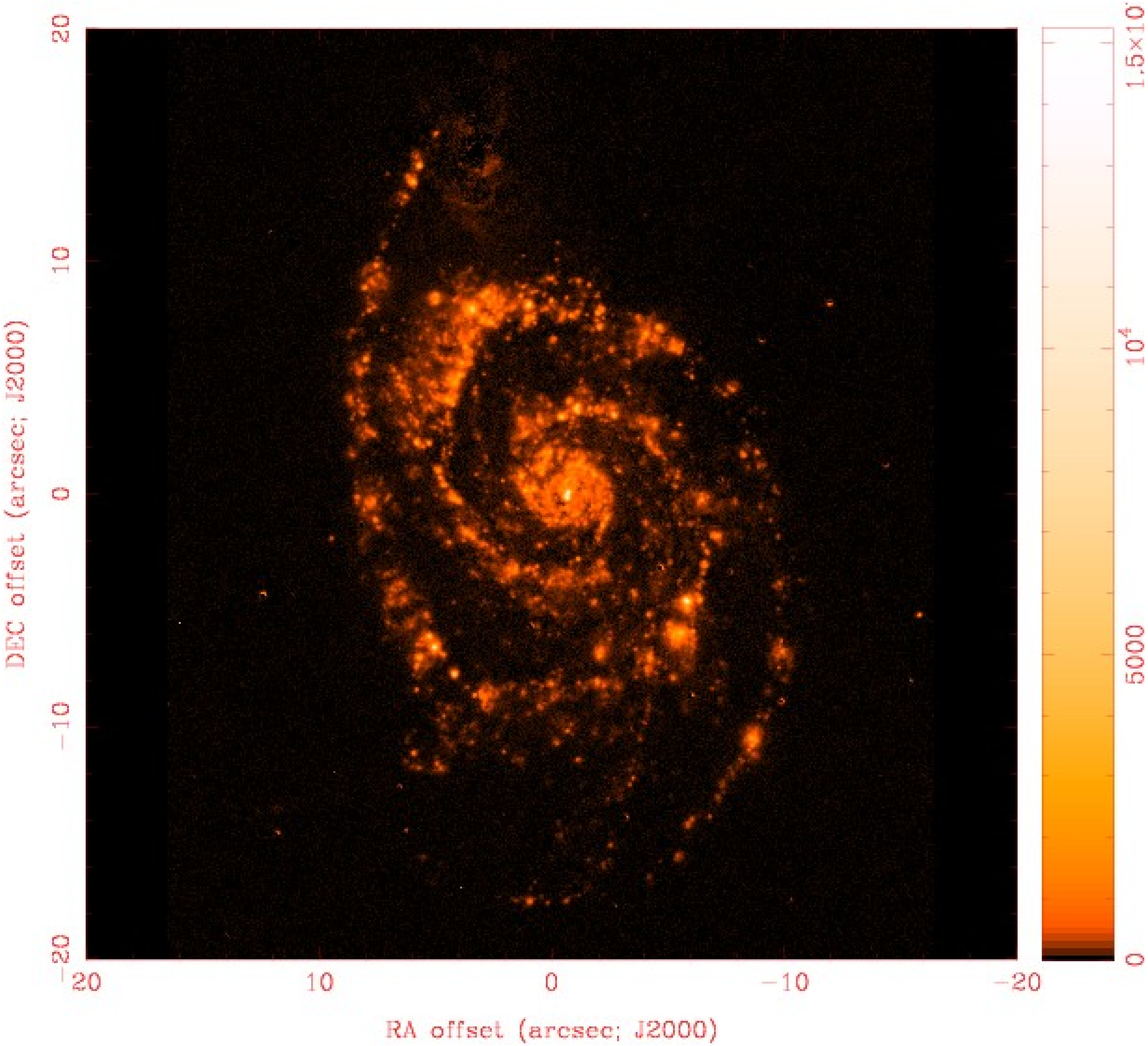
67
400
200
0
−200
−400
V (meters)
−400
−200
G10 47−CONT.TUV
Source: G10.47+0.03
0
00°00′20″
00°00′00″
00°00′20″
00h00m02s
00h00m00s
−00h00m02s
G10 47−CONT−LSB.BEAM
Source: G10.47+0.03
Line: HC3N
Frequency: 106.233301 GHz
Beam: (no clean beam)
Level step: 100 mJy
Box marking: VELOCITY
Channels: [0,0]
schilke@hp175
15−JUL−1999 13:46:38

Bad image



19°51′20″
19°51′40″
19°52′00″
19°52′20″
18h08m40s
18
G10 47−CONT−LSB.lmv
Source: G10.47+0.03
Line: HC3N
19°51′20″
19°51′40″
19°52′00″
19°52′20″
18h08m40s
18h08m38s
G10 47−CONT−LSB.lmv−
Source: G10.47+0.03
Line: HC3N
Frequency: 106.233301 GHz
Beam: 5.81 x 1.76 PA 13°
Level step: 50 mJy/beam
0.53 K
Box marking: VELOCITY
Channels: [0,0]
schilke@hp175
15−JUL−1999 13:45:15

further examples: simulations



Bad (u,v) coverage

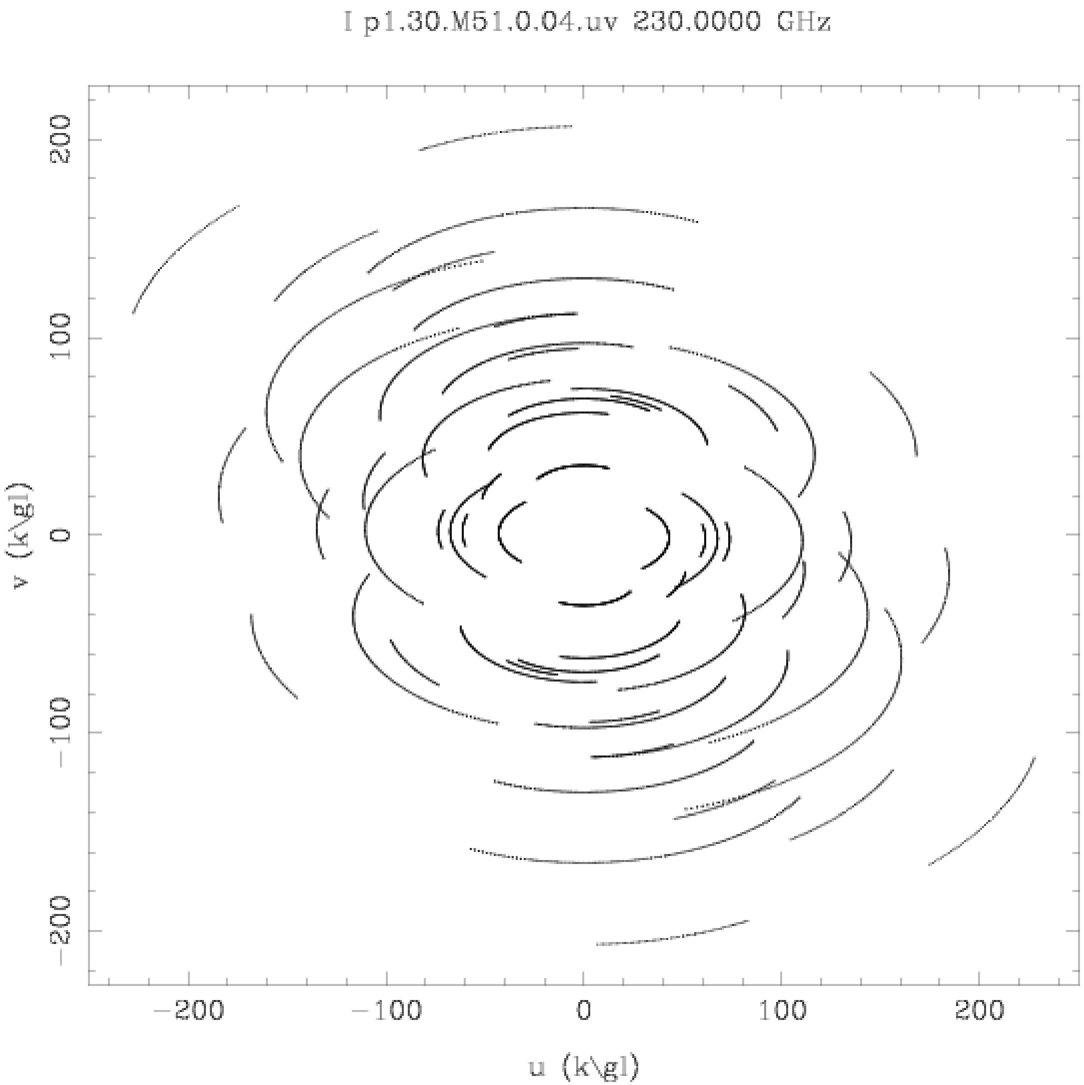
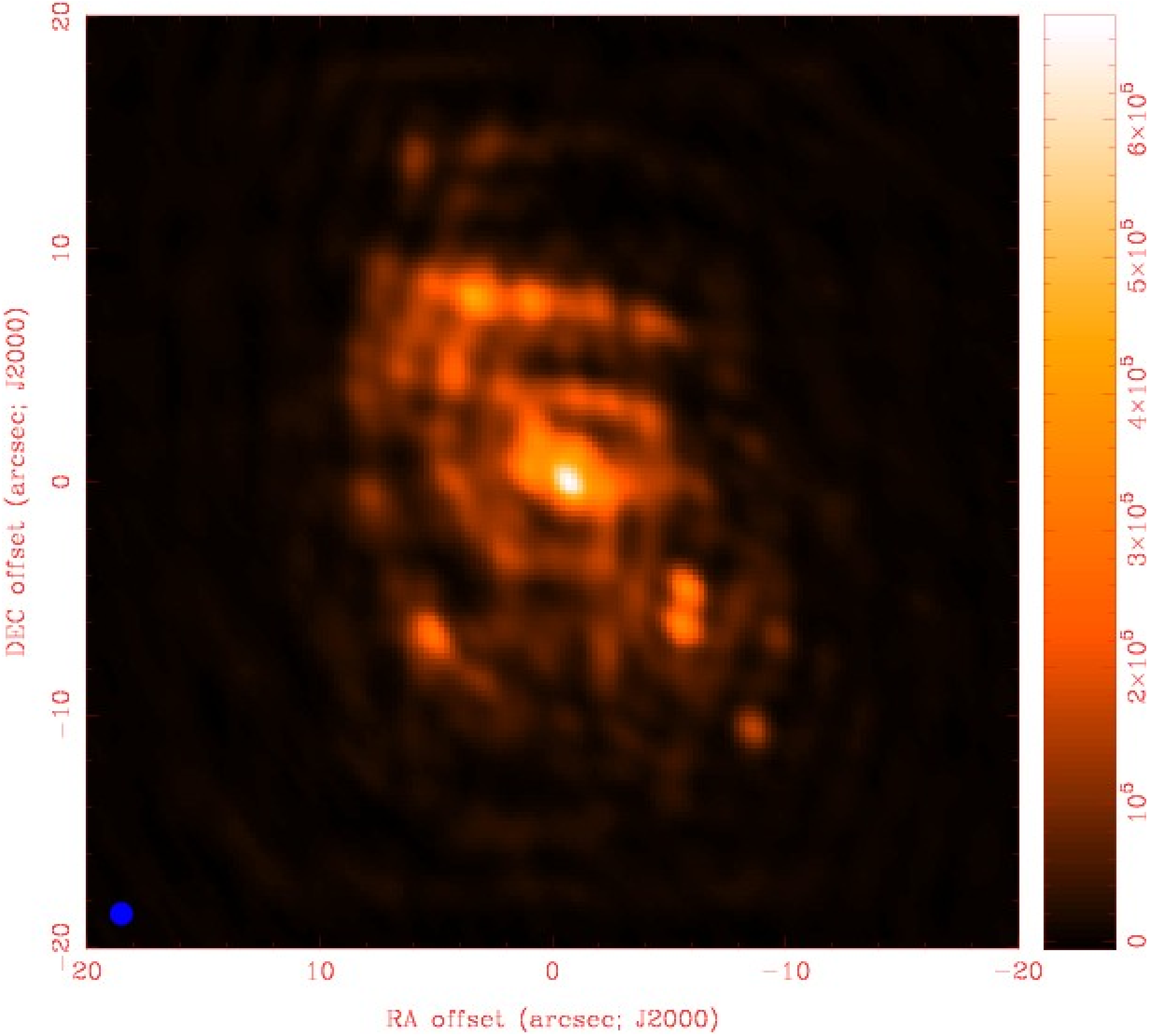
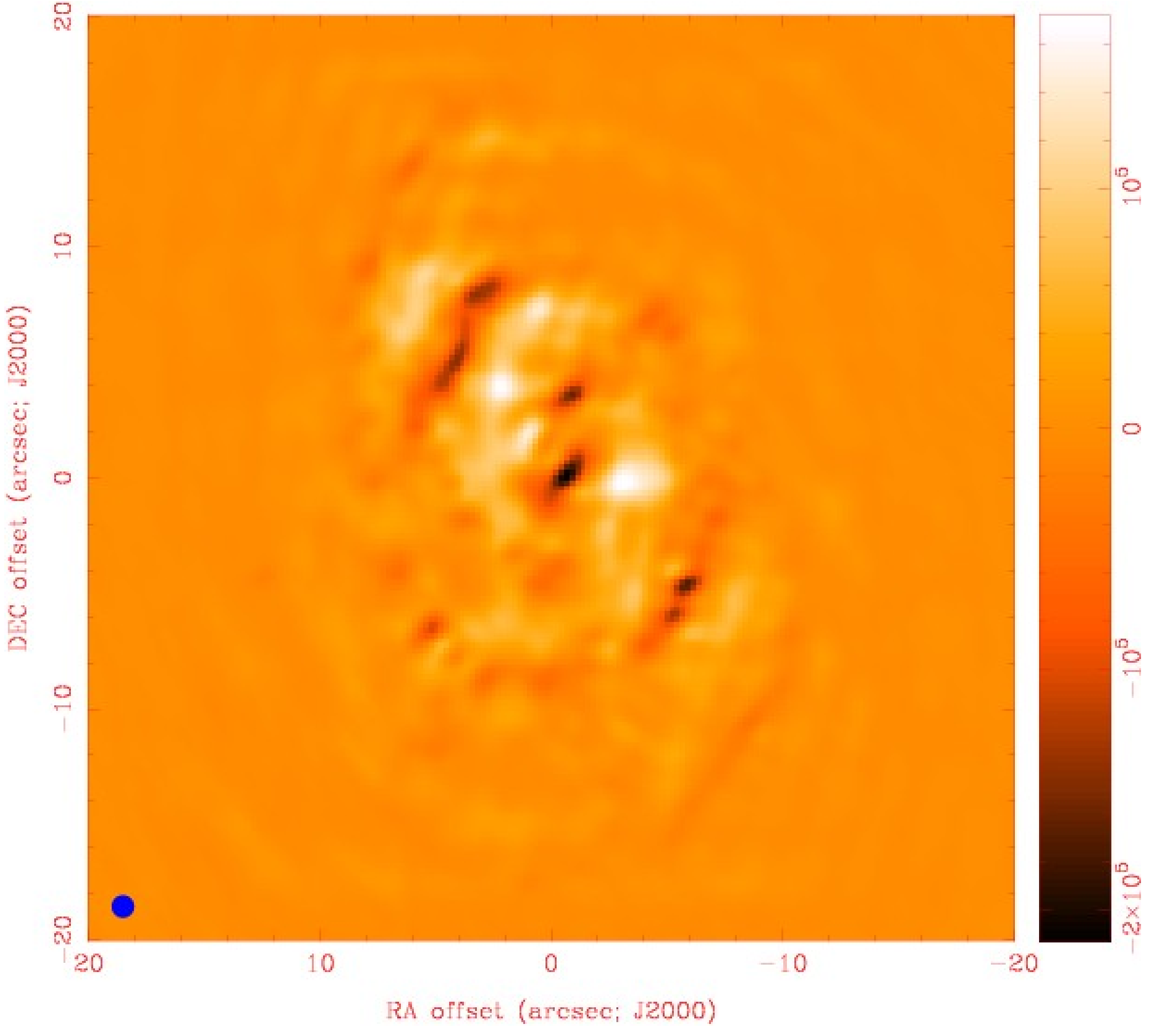


Image (including short spacings)



Residual



20
10
0
−10
−20
DEC offset (arcsec; J2000)
20
10
0
−10
−20
RA offset (arcsec; J2000)
10⁵
0
−10⁵
−2×10⁵

Better (u,v) coverage

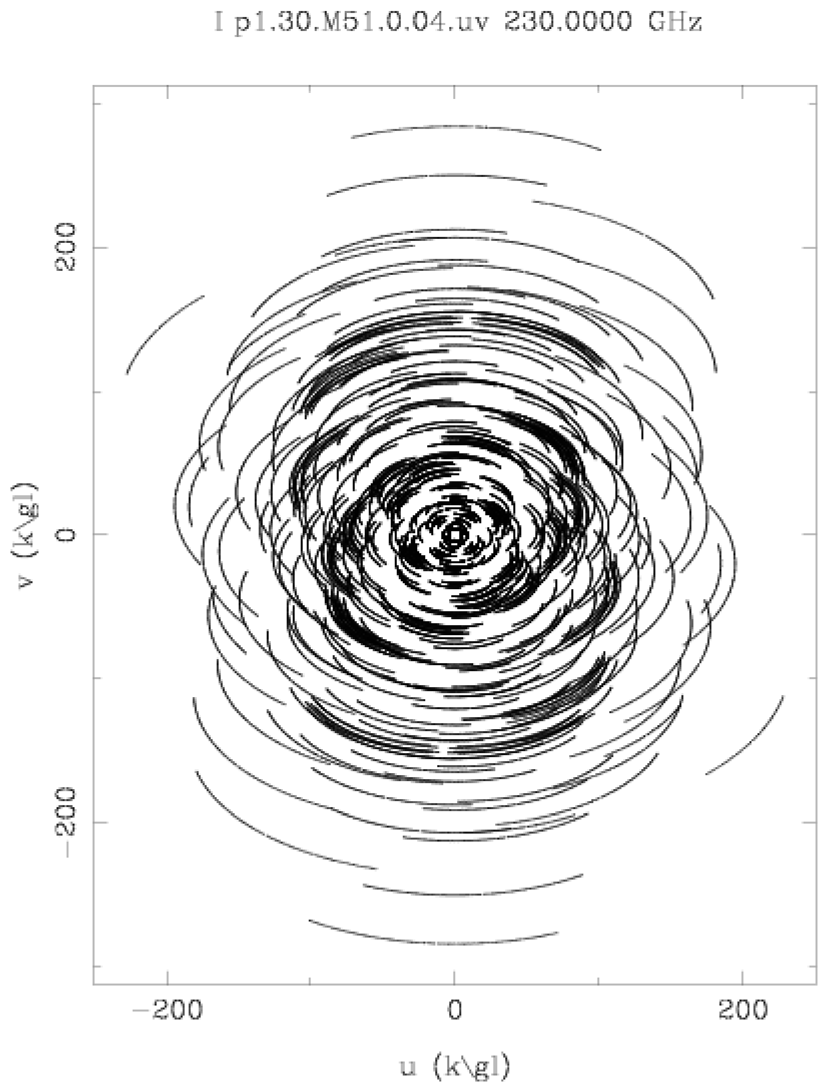
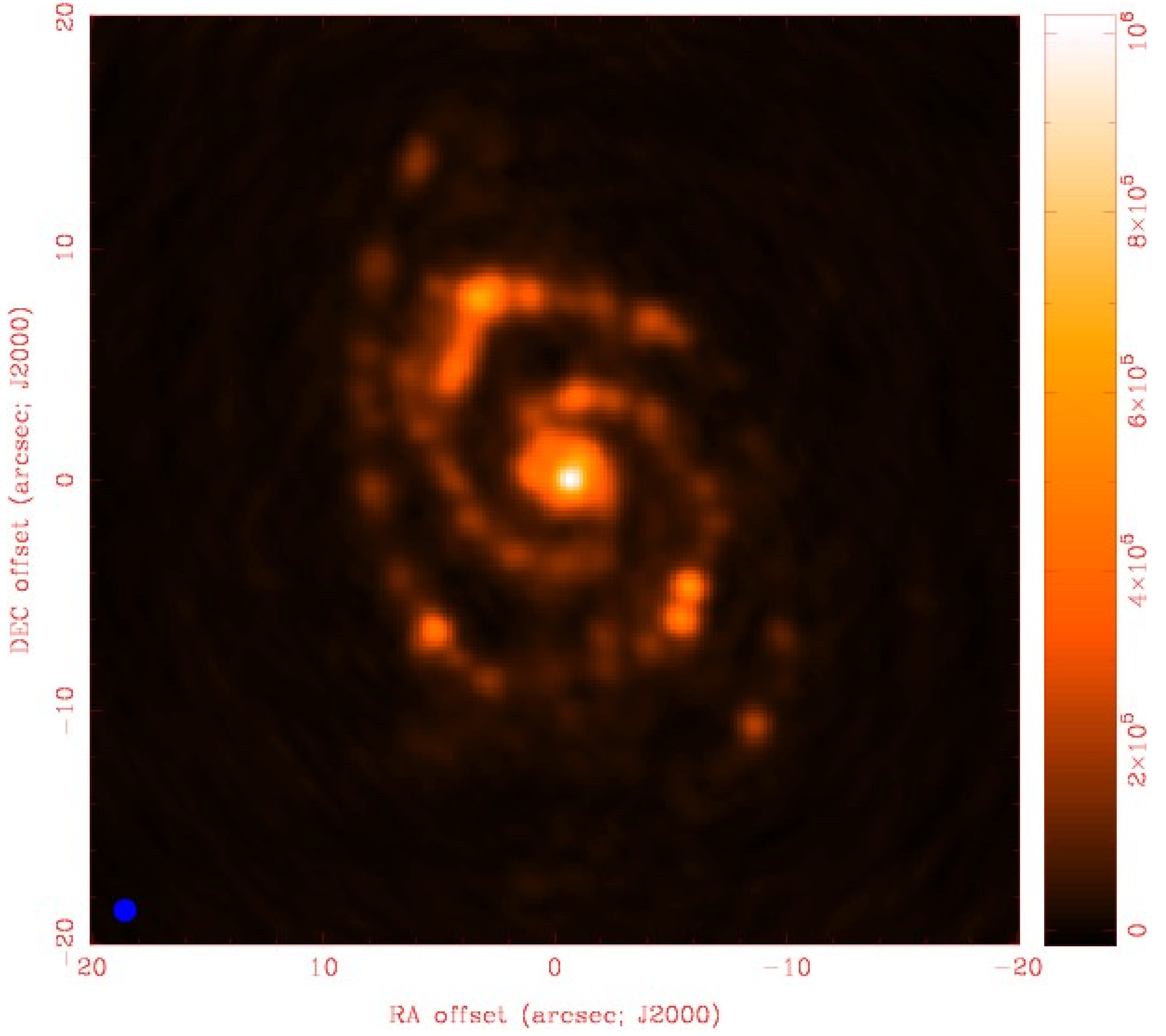
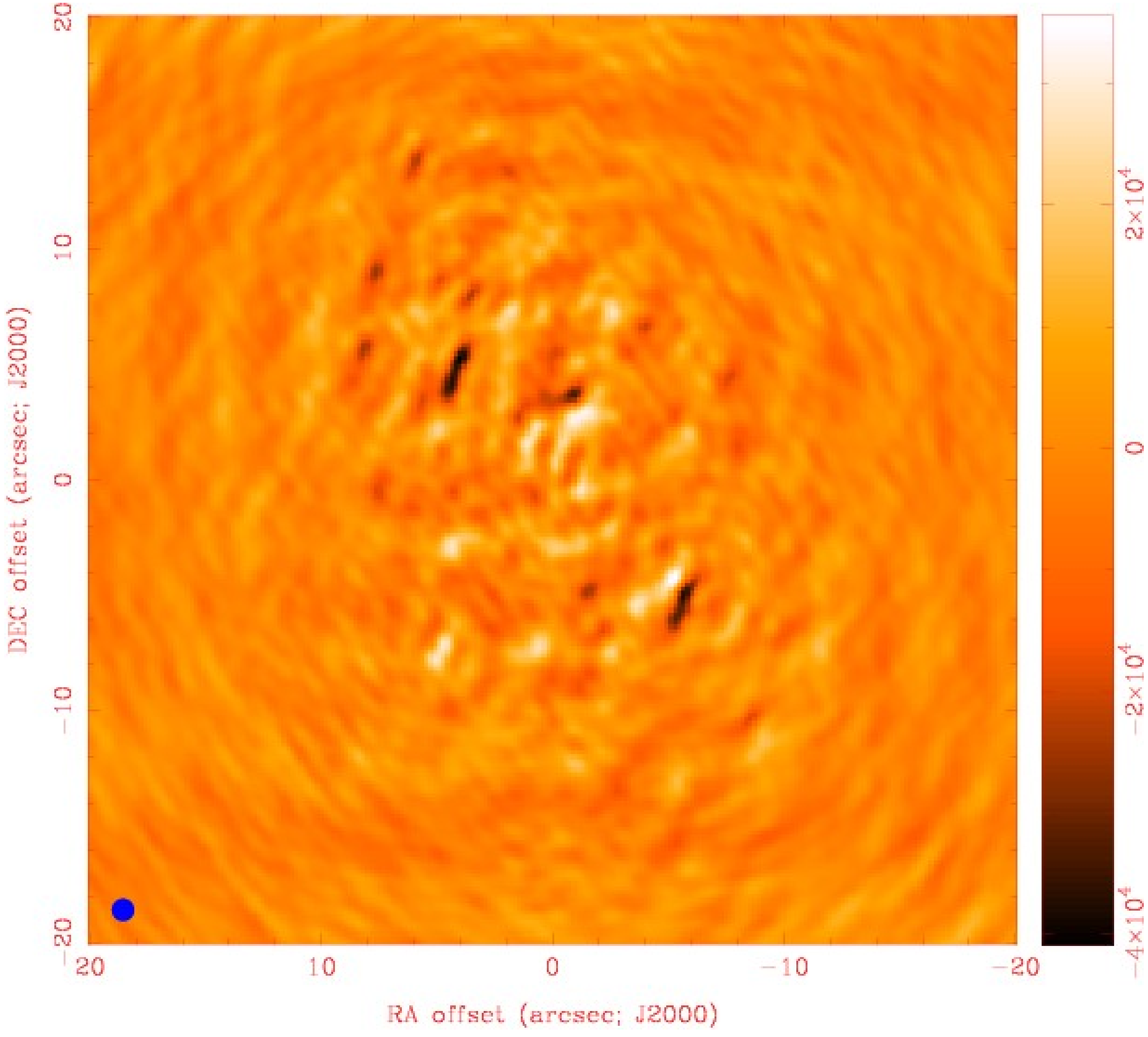


Image (including short spacings)

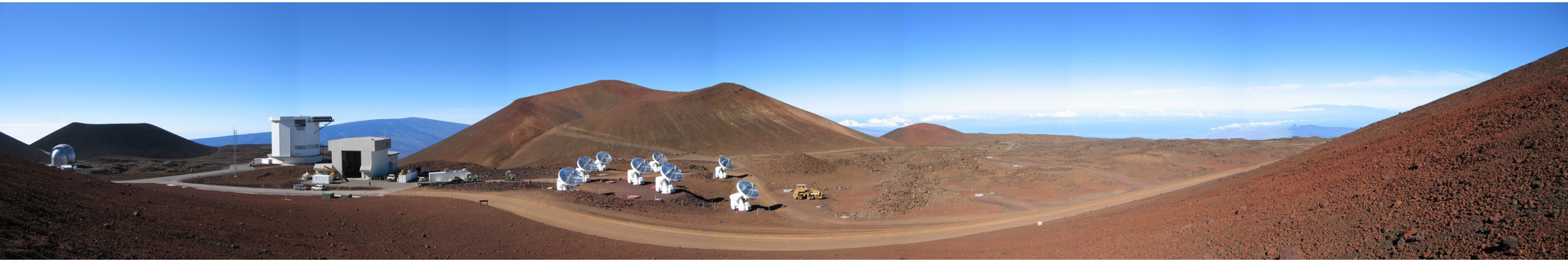


Residual



DEC offset (arcsec; J2000)
20
10
0
−10
−20
20
10
0
−10
−20
RA offset (arcsec; J2000)
2×10^4
0
−2×10^4
−4×10^4

Interferometers



ALMA