

# Global gravity field models from the GPS positions of CHAMP, GRACE and GOCE satellites

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## Summary

### Inversion method

- Data: kinematic orbits, SST high-low (CHAMP, GRACE, GOCE)
- **Acceleration approach**: Newton's second law relates the observed acceleration of the satellite with forces acting on it
- Observed accelerations: numerical 2<sup>nd</sup> derivative of GPS orbits
- Other forces: modelled (e.g. tides) or measured (nongrav. acc.)

### Some original elements of our inversion method

- Aim: simple, straightforward and statistically correct model
- Model is linear in harmonic coefficients, **no a priori** gravity field model is needed, **no regularization** is applied
- **Amplified noise** due to numerical derivative **mitigated** by Generalized Least Squares (linear transformation)
- **Decorrelation** of errors in GPS positions **significantly improves** the accuracy of harmonic **coefficients** by a factor of 2-3
- Separately computed along-track, cross-track and radial solutions merged into combined solution using normal matrices

### Results from real-world data of CHAMP, GRACE, GOCE

- Kinematic orbits of CHAMP and GRACE: 7 years (2003-2009)  
Kinematic orbits of GOCE: 2 months (Nov/Dec 2009)
- **Long-term static gravity field models** (CHAMP, GRACE, GOCE): similar or better quality compared to other published solutions
- **Time variable gravity** (CHAMP, GRACE): mean annual signal clearly shows important hydrological variations on continents
- **Geocentre motion** (GRACE): estimated degree-one coefficients display seasonal variations in accordance with results from other measurement techniques
- Computations performed on ordinary PC up to max. degree 120

## Method of inversion

### (1) Linear regression model

$$\sum SC \times \nabla SSH(r, \theta, \varphi) = d^2r/dt^2 - (a_{LS} + a_{TID} + a_{NG})$$

**Stokes coefficients (SC)** are obtained from observational equations, where:

$\nabla SSH$  ... gradient of solid spherical harmonics

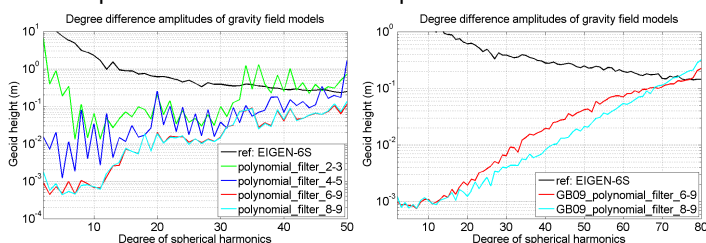
$d^2r/dt^2$  ... observed accelerations

$a_{LS}$  ... lunisolar effects;  $a_{TID}$  ... solid Earth and ocean tides

$a_{NG}$  ... acc. of nongravitational origin (drag, radiation pressures)

### (2) Numerical approximation to the second derivative

- Digital filter of the second derivative is an approximation to the analytical operation. Solutions may differ significantly depending on the particular choice of the filter parameters.

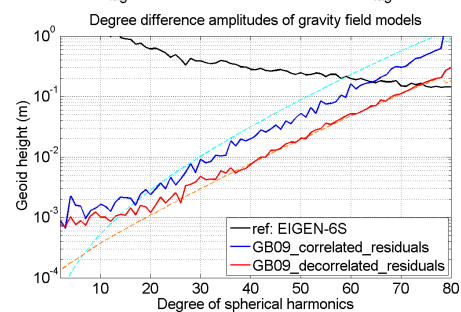
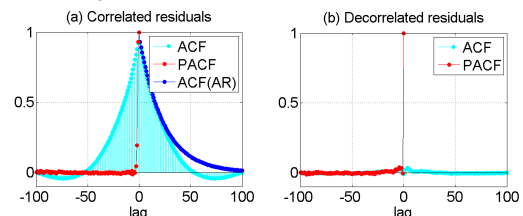


### (3) Mitigation of noise amplification

- Problem: Numerical derivative amplifies noise in GPS positions
- Solution: Generalized least squares (GLS) → application of GLS leads to **linear transformation** of model (1)

### (4) Decorrelation of noise in GPS positions

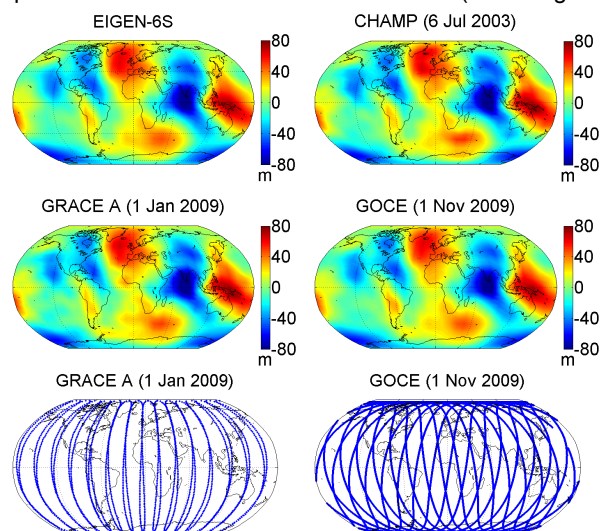
- Problem: GPS positions have correlated errors
- Solution: Sample autocorrelation function (ACF) and especially partial autocorrelation function (PACF) indicate suitability of autoregressive model (AR) to represent correlation structure
- **Decorrelation** of residuals **improved** the **accuracy** of harmonic coefficients by a **factor of 2-3**
- Decorrelation again defines a linear transformation of model (1)



## Results for real orbits

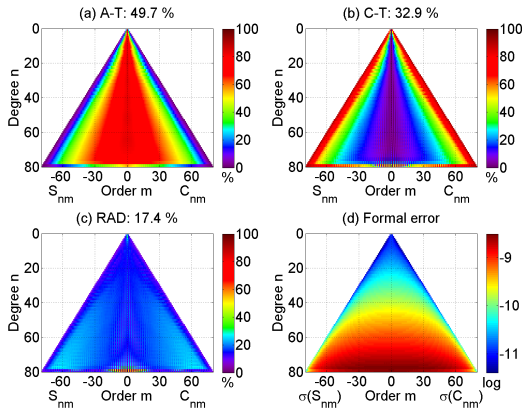
### (5) Gravity field from one day of real data

- Model (1) is linear in SC, no **a priori gravity field** model is used. After applying two linear transformations (3) and (4), SC are obtained directly in one step using the **ordinary least squares**.
- Graphs show reasonable results for real data (max. degree 10)



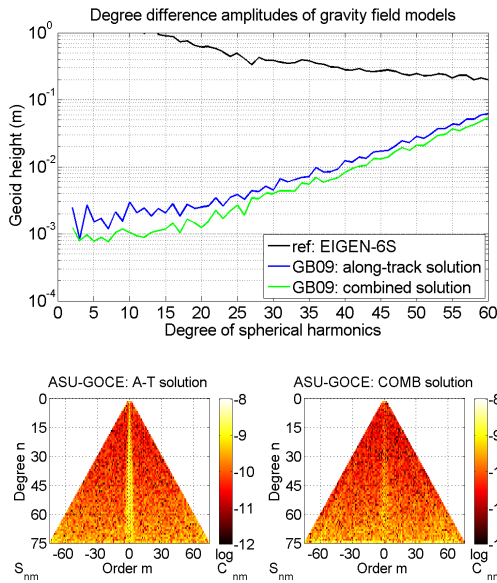
## (6) Along-track, cross-track and radial solutions combine

- We found it better first to compute individual solutions for **along-track** (A-T), **cross-track** (C-T) and **radial** (RAD) directions.
- Then we obtained a **combined solution** using normal matrices.
- Relative contribution of the along-track component to the combined solution is 50 percent on average.



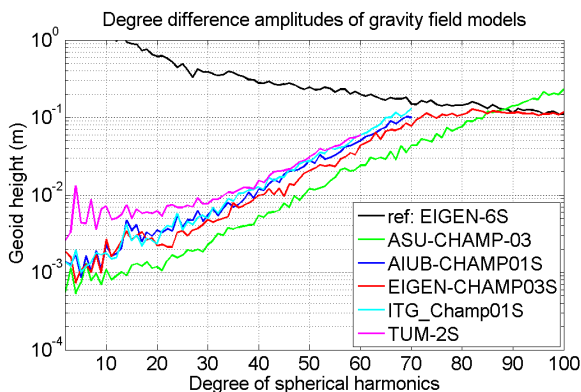
## (7) Along-track solution vs. combined solution

- Systematically, individual **along-track** solutions give **worse results** compared to combined solutions.
- Polar gap of **GOCE**: the combined solutions give better near-zonal coefficients than along-track ones.



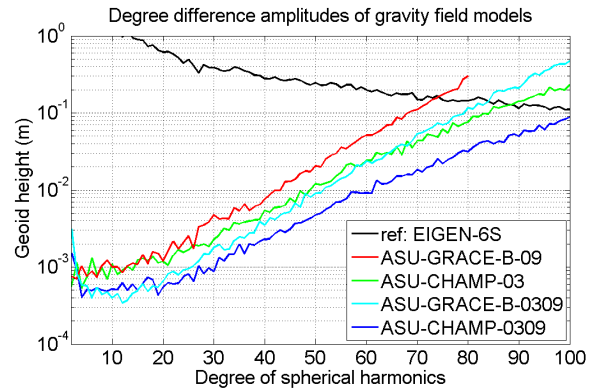
## (8) Gravity field model from orbits of CHAMP in 2003

- This CHAMP solution was computed by many groups (see <http://icgem.gfz-potsdam.de/>)
- Satisfactory behaviour of our solution (ASU-CHAMP-03) is also due to improved processing of kinematic orbits by AIUB



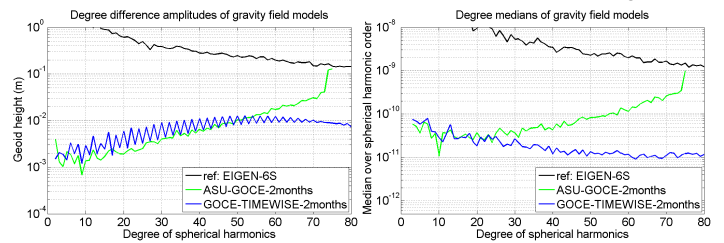
## (9) Seven-year solutions from CHAMP and GRACE orbits

- Apart from physical causes of difference (e.g. mean altitude: CHAMP...400 km, GRACE...500 km), also the quality of GPS data and parameters of the method may play the role (e.g. (2))



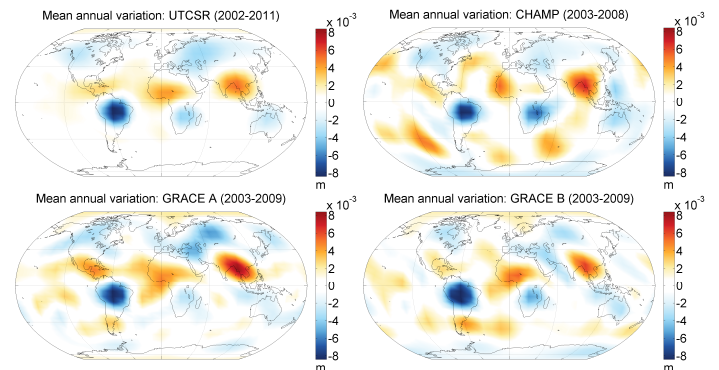
## (10) First results for GOCE

- This is our preliminary result compared to the ESA solution, whose long-wave part is supposed to be not regularized
- In the ESA solution, SGG data start to dominate from degree 25



## (11) Time variable gravity from GPS orbits (CHAMP, GRACE)

- The acquired average annual signal shows clearly well-known **continental areas** with **important hydrological variations**



## (12) Geocentre motion

- We tried to estimate the **degree-one harmonic coefficients**, which correspond to the geocentre motion
- Our orbit-based seasonal variations are **in accordance with results from other measurement techniques**

| Data set             | x<br>amp(mm) | x<br>phase(deg) | y<br>amp(mm) | y<br>phase(deg) | z<br>amp(mm) | z<br>phase(deg) | time span |
|----------------------|--------------|-----------------|--------------|-----------------|--------------|-----------------|-----------|
| SLR: UTCSR           | 3.0 ± 1.2    | 53.9 ± 22.3     | 2.5 ± 1.0    | -39.7 ± 21.7    | 4.6 ± 1.9    | 35.4 ± 23.4     | 2005-2011 |
| Rietbroek et al 2011 | 2.0 ± 0.5    | 58.9 ± 14.5     | 3.6 ± 0.4    | -40.6 ± 6.3     | 3.6 ± 0.7    | 16.1 ± 11.6     | 2005-2009 |
| Swenson et al 2008   | 1.6 ± 0.3    | 99.7 ± 11.1     | 1.2 ± 0.4    | -88.8 ± 16.8    | 2.1 ± 0.3    | 96.0 ± 7.2      | 2005-2010 |
| ASU: GA0509          | 1.2 ± 0.8    | 103.3 ± 37.1    | 1.3 ± 0.8    | -21.6 ± 36.4    | 4.0 ± 2.6    | 11.7 ± 37.3     | 2005-2009 |
| ASU: GB0509          | 1.1 ± 0.7    | 68.0 ± 39.1     | 1.4 ± 0.9    | -19.7 ± 35.6    | 4.6 ± 2.9    | 7.7 ± 36.2      | 2005-2009 |

## Acknowledgments

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